Basic VNS algorithms for solving the pollution location inventory routing problem

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Abstract. This work presents a new variant of the Location Inventory Routing Problem (LIRP), called Pollution LIRP (PLIRP). The PLIRP considers both economic and environmental impacts. A Mixed Integer Programming (MIP) formulation is employed and experimental results on ten randomly generated small-sized instances using CPLEX are reported. Furthermore, it is shown that, CPLEX could not compute any feasible solution on another set of ten randomly generated medium-sized instances, with a time limit of five hours. Therefore, for solving more computationally challenging instances, two Basic Variable Neighborhood Search (BVNS) metaheuristic approaches are proposed. A comparative analysis between CPLEX and BVNS on these 20 problem instances is reported.

Keywords: Variable Neighborhood Search · Location Inventory Routing Problem · Green Logistics.

1 Introduction

In recent years, the efforts to manage the environmental impacts of the logistic activities have been increased. One of the major environment challenges is the global warming. The carbon dioxide (CO_2) emissions are highlighted as its main cause (2; 5). Transportation has been mentioned as the logistic activity with the highest contribution to (CO_2) emissions (2; 11). Also, the combined environmental impact of location-routing activities (10) and inventory-routing activities (2) has already been studied.

More specifically, the amount of the emitted (CO_2) gasses is proportionate to the amount of the consumed fuel (6). Based on that fact, companies can either adopt energy efficient vehicles or re-optimize their logistic decisions by taking into account factors affecting the fuel consumption (2), or even adopt a hybrid strategy.

From an economic perspective, the simultaneous tackling of strategic, tactical and operational decisions ensured the efficient performance of the supply chain.

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The Location Inventory Routing Problem (LIRP) integrates these three decisions (8; 13). However, there is a lack of research about the environmental-related variants of the LIRP. A sustainable closed-loop LIRP proposed by (12) where economic, environmental and social impacts were considered. They formulated a multi-objective stochastic programming model for describing the problem.

In this work, a new green variant of the LIRP, the Pollution Location Inventory Routing Problem (PLIRP) is presented. A Mixed Integer Programming (MIP) model is presented. In order to solve medium- and large-scaled instances, Basic Variable Neighborhood Search metaheuristic algorithms are developed. The remainder of this work is organized as follows. Section 2 describes the problem and provides its mathematical formulation. In Section 3, the proposed solution approach is presented, followed by experimental results in Section 4. Finally, Section 5 concludes this work and outlines future directions.

2 Problem statement

This work extends the LIRP presented in (13) by considering fuel consumption and (CO_2) emissions costs, that are influenced by distance, load, speed and vehicles characteristics. The mathematical formulation of the PLIRP integrates the MIP models presented in (13) and (2). The notations of the proposed model are given in Table 1.

Table 1. PLIRP model variables and parameters.

Notation	Explanation
f_j	fixed opening cost of depot j
y_j	1 if j is opened; 0 otherwise
C_j	storage capacity of depot j
z_{ij}	1 if customer i is assigned to depot j; 0 otherwise
h_i	unit inventory holding cost of customer i
Q_k	loading capacity of vehicle k
d_{it}	period variable demand of customer i
x_{ijkt}	1 if node j is visited after i in period t by vehicle k
q_{ikt}	product quantity delivered to customer i in period t by vehicle k
w_{itp}	quantity delivered to customer i in period p to satisfy its demand in period t
c_{ij}	travelling cost of locations pair (i, j)
a_{vikt}	load weight by travelling from node v to the customer i with vehicle k in period t
$zz_{v_1 v_2 ktr}$	1 if vehicle k travels from node $v1$ to $v2$ in period t with speed level r
s_r	the value of the speed level r

Table 2 describes the vehicles' parameters and gives their fixed values.

Table 2. Vehicles' parameters.

Parameter	Explanation	Value
ϵ	fuel-to-air mass ratio	1
g	gravitational constant (m/s^2)	9.81
ρ	air density (kg/m^3)	1.2041
CR	coefficient of rolling resistance	0.01
η	efficiency parameter for diesel engines	0.45
f_c	unit fuel cost (e/L	0.7382
f_e	unit CO_2 emission cost e/kg	0.2793
σ	CO_2 emitted by unit fuel consumption (kg/L)	2.669
$HVDF$	heating value of a typical diesel fuel (kj/g)	44
ψ	conversion factor (g/s to L/s)	737
θ	road angle	0
τ	acceleration (m/s^2)	0
CW_k	curb weight (kg)	3500
EFF_k	engine friction factor $(kj/rev/L)$	0.25
ES_k	engine speed (rev/s)	39
ED_k	engine displacement (L)	2.77
CAD_k	coefficient of aerodynamics drag	0.6
FSA_k	frontal surface area (m^2)	9
$VDTE_k$	vehicle drive train efficiency	0.4

It should be highlighted that, the values of parameters f_c and f_e are the average price of the petrol prices in 40 European countries, taken from the site `www.globalpetrolprices.com` in 26th of February in 2018. The value of parameter CW_k can refer to (9). The rest of the parameters' values are taken by (2).

In order to simplify some parts of the objective function, due to the fuel consumption, the following formulas are utilized.

- $\lambda = \frac{HVDF}{\psi}$
- $\gamma_k = \frac{1}{1000VDTE\eta}$
- $\alpha = \tau + gCR\sin\theta + gCR\cos\theta$
- $\beta_k = 0.5CAD\rho FSA_k$

Thus, the mathematical model of the PLIRP is as follows:

$$\min \sum_{j\in J} f_j y_j + \sum_{i\in I} h_i \sum_{t\in H}\left(\frac{1}{2}d_{it} + \sum_{p\in H, p<t} w_{itp}\,(t-p) + \sum_{p\in H, p>t} w_{itp}\,(t-p+H)\right)$$
$$+ \sum_{i\in V}\sum_{j\in V}\sum_{t\in H}\sum_{k\in K} c_{ij}x_{ijkt} + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{t\in H}\left\{\lambda\left(f_c + (f_e\sigma)\right)\left(\sum_{r\in R}\frac{(zz_{ijktr}EFF_kES_kED_kc_{ij})}{s_r}\right.\right.$$
$$\left.\left. + \left(\alpha\gamma_k\left(CW_kx_{ijkt} + a_{ijkt}\right)c_{ij}\right) + \left(\beta_k\ \gamma_k \sum_{r\in R}\left(s_r\ zz_{ijktr}\right)^2\right)\right)\right\} \tag{1}$$

Subject to

$$\sum_{r\in R} zz_{ijktr} = 0 \quad \forall i,j \in V, \forall k \in K, \forall t \in H \tag{2}$$

$$\sum_{i\in V} a_{ijkt} - \sum_{i\in V} a_{jikt} = q_{jkt}PW \quad \forall j \in I, \forall k \in K, \forall t \in H \tag{3}$$

$$\sum_{j\in V} x_{ijkt} - \sum_{j\in V} x_{jikt} = 0 \qquad \forall i \in V, \forall k \in K,\ \forall t \in H \tag{4}$$

$$\sum_{j\in V}\sum_{k\in K} x_{ijkt} \leq 1 \quad \forall t \in H,\ \forall i \in I \tag{5}$$

$$\sum_{j\in V}\sum_{k\in K} x_{jikt} \leq 1 \quad \forall t \in H,\ \forall i \in I \tag{6}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijkt} \leq 1 \quad \forall k \in K,\ \forall t \in H \tag{7}$$

$$x_{ijkt} = 0 \quad \forall i,j \in J,\ \forall k \in K,\ \forall t \in H,\ i \neq j \tag{8}$$

$$\sum_{i\in I} q_{ikt} \leq Q_k \quad \forall k \in K,\ \forall t \in H \tag{9}$$

$$\sum_{j\in J} z_{ij} = 1 \quad \forall i \in I \tag{10}$$

$$z_{ij} \leq y_j \quad \forall i \in I,\ \forall j \in J \tag{11}$$

$$\sum_{i\in I}\left(z_{ij}\sum_{t\in H} d_{it}\right) \leq C_j \quad \forall j \in J \tag{12}$$

$$\sum_{u\in I} x_{ujkt} + \sum_{u\in V\setminus\{i\}} x_{iukt} \leq 1 + z_{ij} \quad \forall i \in I,\ \forall j \in J,\ \forall \mathrm{k} \in \mathrm{K}, \forall t \in H \tag{13}$$

$$\sum_{i\in I}\sum_{k\in K}\sum_{t\in H} x_{jikt} \geq y_j \quad \forall j \in J \tag{14}$$

$$\sum_{i\in I} x_{jikt} \leq y_j \quad \forall j \in J,\ \forall k \in K,\ \forall t \in H \tag{15}$$

$$\sum_{p\in H} w_{itp} = d_{it} \quad \forall i \in I,\ \forall \mathrm{t} \in \mathrm{H} \tag{16}$$

$$\sum_{t \in H} w_{itp} = \sum_{k \in K} q_{ikp} \quad \forall i \in I,\ \forall p \in H \tag{17}$$

$$q_{ikt} \leq M \sum_{j \in V} x_{ijkt} \quad \forall i \in I,\ \forall t \in H,\ \forall k \in K \tag{18}$$

$$\sum_{j \in V} x_{ijkt} \leq M q_{ikt} \quad \forall i \in I,\ \forall t \in H,\ \forall \mathrm{k} \in \mathrm{K} \tag{19}$$

$$\mathrm{x}_{ijkt} \in \{0,1\} \quad \forall i \in I,\ \forall j \in J,\ \forall t \in H,\ \forall k \in K \tag{20}$$

$$y_j \in \{0,1\} \quad \forall j \in J \tag{21}$$

$$z_{ij} \in \{0,1\} \quad \forall i \in I,\ \forall j \in J \tag{22}$$

$$q_{ikt} \leq \min \left\{ Q_k, \sum_{p \in H} d_{ip} \right\} \quad \forall i \in I,\ \forall j \in J\ , \forall k \in K \tag{23}$$

$$w_{itp} \leq d_{ip} \quad \forall i \in I,\ \forall t, p \in H \tag{24}$$

The objective function minimizes the sum of facilities opening costs, inventory holding costs, general routing costs and fuel consumption and CO_2 emissions costs. Constraints 2 impose that, only one speed level will be assigned to a vehicle traveling between two nodes in the selected time period. Constraints 3 declare that, the total weight of the incoming flow of product to a selected customer minus the total weight of the outcoming product flow of that customer equals the product weight delivered to that customer in the selected time period with the selected vehicle. Also, they operate as subtour elimination constraints. Constraints $2-9$ are related to the routing decisions. As an example Constraints 8 ensure that, a selected vehicle in a selected time period will not travel between two depots. Constraints $10-15$ guarantee the feasibility of the location decisions. For example, Equations 10 and 11 force a customer to be allocated to a depot, only if that depot is marked as opened. Finally, Constraints $11-19$ force the feasibility of the inventory decisions. For instance, Constraints 16 guarantee the satisfaction of a selected customer's demand over the time horizon.

3 Solution Method

Due to the high computational complexity of the PLIRP, two Basic Variable Neighborhood Search metaheuristic algorithms are proposed for solving medium and large scale problem instances.

3.1 Construction Heuristic

Initially, a feasible solution is built by applying a three phase constructive heuristic. In the first phase, a minimum cost criterion procedure is applied for selecting the depots to be opened. Then, the allocation of customers is sequentially scheduled. More specifically, if the total demand of a selected customer does not violate the remaining capacity of the selected opened depot, the customer is allocated to that depot. Otherwise, the customer is allocated to the next opened and capable to service him depot. Finally, the routes are built by applying a random insertion method. It should be clarified that, in this initial solution the scheduled product quantity to be delivered to each customer at each period satisfy its demand for the considered period.

3.2 Basic VNS

The Variable Neighborhood Search is a trajectory-based metaheuristic framework which interchanges two main phases (7). The first one is the intensification phase, where a local optimum solution is obtained and the second one is the diversification phase. In the last phase the current solution is perturbed for escaping local optimum points. VNS has gained popularity in recent years due to its simplicity and performance (3). In this work two local search operators are used both in improvement and shaking phase. These neighborhood structures are the following.

– **Inter-route Exchange**. Solutions in this neighborhood are obtained by exchanging two customers located in different routes. These routes could be allocated either on the same or different depots. In the second case, inventory replenishment rescheduling may need to be applied. Figures 1 and 2, illustrate the inter-route exchange move in routes allocated to the same depot. In the first time period an exchange between the third and fourth customers is applied, while in the second and third periods the exchanged customers are the pairs $(1, 4)$ and $(3, 5)$, respectively.

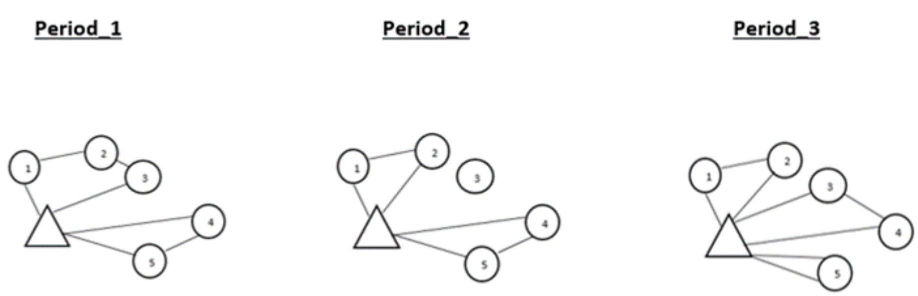

Fig. 1. Routes from the same depot for each time period before the application of the inter-route exchange move.

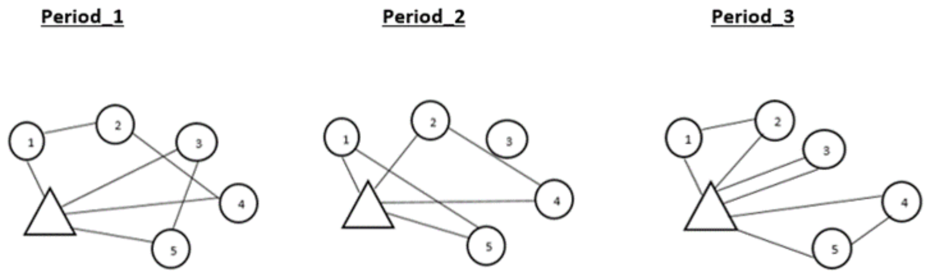

Fig. 2. Routes from the same depot for each time period after the application of the inter-route exchange move.

When the routes are allocated to different depots, the exchange move is applicable only if the two customers are serviced in the same time periods. Figures 3, 4, and 5 illustrate the inter-route exchange between the customers two and three. It is also assumed that, the product quantity delivered to customer three in the third time period exceeds the capacity of the vehicle currently servicing the customer two. Consequently, a replenishment rescheduling is applied as depicted in Figure 5.

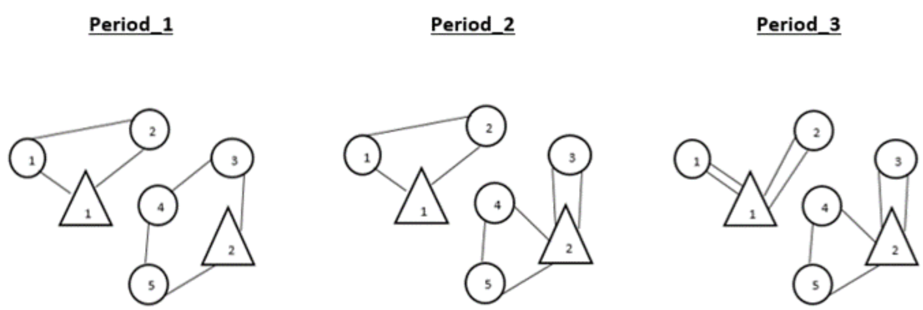

Fig. 3. Routes from different depots for each time period before the application of the inter-route exchange move.

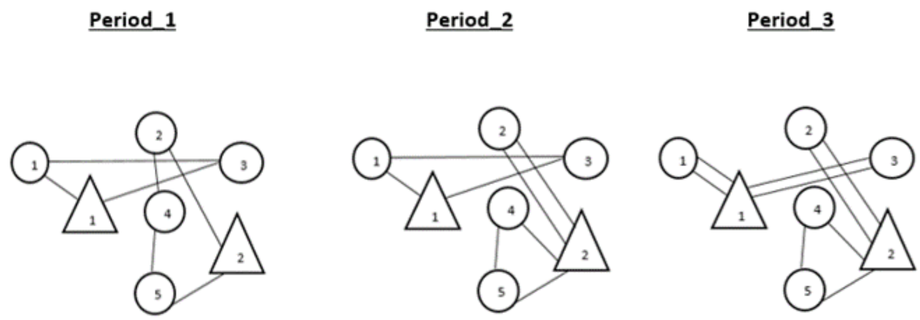

Fig. 4. Routes from different depots for each time period after the application of the inter-route exchange move.

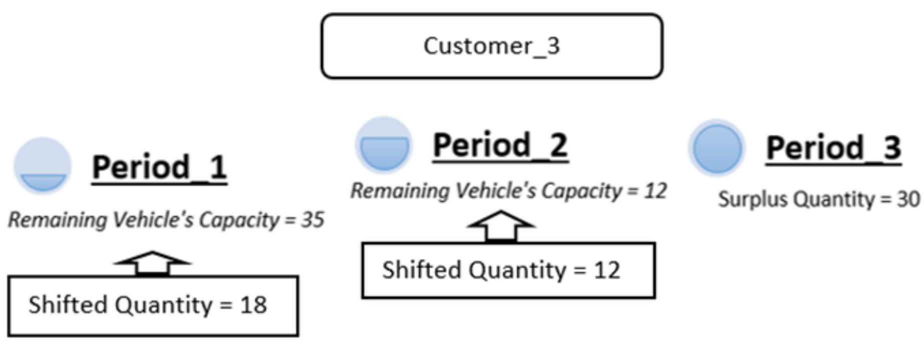

Fig. 5. Shifting the surplus quantity of product of the customer 3 from the third period to other(s).

– **Opened-Closed Depots Exchange**. This neighborhood exchanges a closed depot with an opened one. In each route, allocated to the currently opened depot, a routing reordering procedure is applied.

Figure 6 illustrates an instance of the opened-closed depots' exchange move. More specifically, the first (opened) depot swaps with the second (closed) depot and reordering occurred in the two routes.

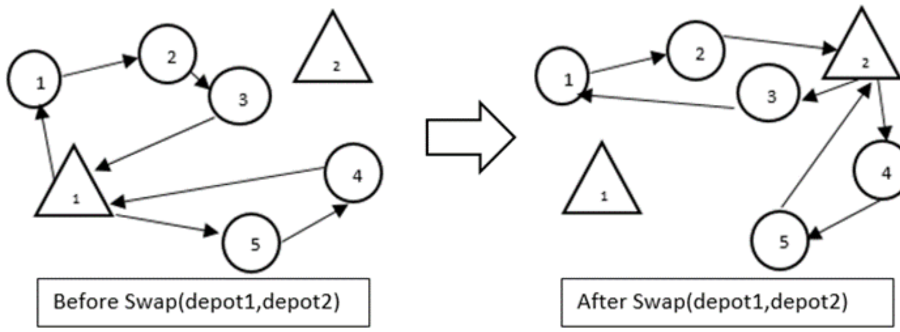

Fig. 6. An illustrated example of the opened-closed depots' exchange move.

It should be mentioned that, the first improvement search strategy is selected for the inter-route exchange local search operator because it is divided into two sub-moves and consequently it is a high complexity move. However, the opened-closed depots exchange local search operator is applied with the best improvement strategy, due to the high impact of the location decisions on the total cost and the low complexity of the move. The pseudocode of the proposed solution approach is provided in Algorithm 1.

Algorithm 1 Basic VNS

```
procedure BVNS(k_max, max_time)

    S ← Construction_Heuristic

    while time ≤ max_time do
        for each neighborhood structure l do
            for k ← 1, k_max do

                S' ← Shake(S, k)
                S'' ← Local_Search(S', l)

                if f(S'') < f(S) then
                    S ← S''
                end if

            end for
        end for
    end while

    Return S
```

Speed Selection Procedure (SSP) examines which speed level has the highest fuel cost decrease for each depot-customer and customer-customer pair in the current solution. It can be used within BVNS and employed after the execution of each local search operator. This version of BVNS is called BVNS_SSP and its pseudocode is summarized at Algorithm 2.

Algorithm 2 Basic VNS with SPP

```
procedure BVNS_SSP(k_max, max_time)

  S ← Construction_Heuristic

  while time ≤ max_time do
    for each neighborhood structure l do
      for k ← 1, k_max do

        S' ← Shake(S, k)
        S'' ← Local_Search(S', l)
        S' = Speed_Selection(S'')

        if f(S') < f(S) then
          S ← S''
        end if

      end for
    end for
  end while

  return S
```

In the *Shake* procedure a randomly selected neighborhood structure is applied k times in a current solution, while the *Local_Search* procedure applies the operator specified by l in an incumbent solution both in BVNS and BVNS_SSP.

4 Numerical Results

4.1 Computing Environment & Parameter Settings

The proposed algorithms were implemented in Fortran. The computational experiments ran on a desktop PC running Windows 7 Professional 64-bit with an Intel Core i7-4771 CPU at 3.5 GHz and 16 GB RAM, using Intel Fortran compiler 18.0 with optimization option /O3. The time limit of 60s was set as the maximum execution time and experimentally kmax is set to four. The mathematical formulation was modeled in GAMS (GAMS 24.9.1) and the problem instances were solved with CPLEX 12.7.1.0 solver with specified time limits (2h for the small-sized instances and 5h for the medium-sized instances). CPLEX ran in the same computing environment with Intel Fortran compiler.

4.2 Computational results

This subsection summarizes the results of the computational tests performed on 20 randomly generated instances, in order to examine the performance of the proposed algorithms. The problem instances are divided into two classes, small-sized instances (less than 20 customers) and medium-sized instances (customers between 20 and 60). Their format follows the format of the instances presented in (13).

Table 3 summarizes the results obtained by the CPLEX solver, BVNS and BVNS with SSP procedure. More specifically, the first column provides the names of the instances. In the second column the results obtained by CPLEX are given, while columns three and five show the average results achieved by BVNS and BVNS_SSP respectively. Columns four and six provide the solution gap between BVNS and CPLEX results and between BVNS and BVNS_SSP results respectively. Finally, the solution quality gap between the two proposed methods are given in column seven.

Table 3. Average computational results on 10 small-sized PLIRP instances

Instance	CPLEX (a)	BVNS (b)	gap (a-b) %	BVNS_SSP (c)	gap (a-c) %	gap (b-c)
4-8-3	19393.307	28677.4	-47.87	28689.85	-47.94	-0.043
4-8-5	18282.71	19617.67	-7.30	19617.88	-7.30	-0.001
4-10-3	16929.958	17816.68	-5.24	17822.41	-5.27	-0.032
4-10-5	-	23895.69	-	23895.86	-	-0.001
4-15-5	22013.988	23085.68	-4.87	23125.5	-5.05	-0.048
5-12-3	17404.069	25368.14	-45.76	25368.33	-45.76	-0.001
5-15-3	18472.911	28230.62	-52.82	28246.89	-52.91	-0.058
5-18-3	20948.331	38440.93	-83.50	38455.22	-83.57	-0.037
5-18-5	-	19874.93	-	19889.2	-	-0.072
5-20-3	24605.637	25118.77	-2.09	25142.75	-2.18	-0.095
		Average	-31.18	Average	-31.25	-0.051

As it can be seen in Table 3, the CPLEX solver (GAMS) provides 31.18% better solutions than BVNS and 31.25% better solutions than BVNS_SSP. However, the time limit for the CPLEX was set at two hours, while both BVNS and BVNS_SSP execute for 60s. CPLEX is not able to provide any feasible solution for the medium-sized instances, even with a time limit of five hours. Consequently, Table 4 reports the results achieved by BVNS and BVNS_SSP on the set of the ten medium-sized instances.

Table 4. Average computational results on ten medium-sized PLIRP instances

Instance	BVNS (a)	BVNS_SSP (b)	gap (a-b) %
6-22-7	28090.69	28074.74	0.057
6-25-5	22381.44	22267.83	0.508
7-25-5	39928.19	39914.65	0.034
7-25-7	23398.09	23538.9	-0.602
8-25-5	26779.47	26771.7	0.029
8-30-7	36690.87	36687.43	0.009
8-50-5	33536.02	33512.67	0.070
8-65-7	27931	27968.24	-0.133
9-40-7	23162.88	23160.74	0.009
9-55-5	23632.86	23666.88	-0.144
		Average	-0.016

As it is shown in Table 4, the solutions obtained by BVNS are 0.016 % better than those achieved by BVNS_SSP. Table 5 reports the best values achieved by both BVNS and BVNS_SSP in all 20 randomly generated PLIRP instances.

Table 5. Best values on 20 PLIRP instances achieved by BVNS and BVNS_SSP

Instance	BVNS	BVNS_SSP
4-8-3	28670.06	28677.4
4-8-5	19301.08	19617.67
4-10-3	17711.27	17816.68
4-10-5	23902.35	23895.69
4-15-5	23134.46	23085.68
5-12-3	25373.42	25368.14
5-15-3	28272.34	28230.62
5-18-3	38427.35	38440.93
5-18-5	19900.76	19874.93
5-20-3	25100.46	25118.77
6-22-7	28089.34	28074.56
6-25-5	22181.95	21949.7
7-25-5	39926.65	39914.5
7-25-7	23126.79	23204.11
8-25-5	26771.04	26769.51
8-30-7	36467.03	36644.86
8-50-5	33505.66	33461.71
8-65-7	27876.26	27892.72
9-40-7	23039.24	23033.97
9-55-5	23586.27	23606.41

5 Conclusions

This work introduces a new variant of the NP-hard combinatorial optimization problem, known as the Pollution Location Inventory Routing Problem which integrates economic and environmental decisions. A MIP formulation of the PLIRP is presented and the optimization solver CPLEX was used for solving small-sized instances. Because of the high complexity of the PLIRP, CPLEX is not able to find find any feasible solution for medium-sized instances even with a time limit of 5h. For solving more challenging instances, two Basic VNS heuristic algorithms were developed. A future research direction can explore parallel computing techniques (1) for speeding up the solution process. This way, more real-world extensions of this type of problems (e.g., including the utilization of remanufacturing options (4)) can be efficiently addressed.

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