On the impact of different inventory policies on the solution of the Inventory-Routing Problem with strict driving hours' regulations and driving speed limits

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Abstract This work addresses a new Inventory Routing Problem with realistic characteristics, such as strict driving hours' regulations and driving speed limits. According to these realistic assumptions, the time needed to perform a route will be strictly bounded by an upper time limit based on the driving hours' regulations provided by the European Commission. Moreover, the decisions about the speed level selection, for travelling between two network points, will be subject to specific driving speed limits. Moreover, the impact of two inventory policies, the classic (R,Q) replenishment policy and the flexible replenishment policy, on the total supply chain network cost is studied. The consideration of two inventory policies leads to the development of two mixed integer linear programming models. The computational experiments were conducted on random, small-sized, problem cases, using the state-of-the-art solver, Gurobi. The impact of the two inventory policies on the supply chain network is investigated through a numerical analysis.

1 Introduction

The integration of supply chain activities has been highlighted as an expedient approach to achieve high business performance (4; 15). Two key supply chain activities are the distribution and the inventory control (32). The integration of these two activities has been thoroughly studied in the literature (11; 20; 28). The investigation of this integration has been proceeded through the study of a complex combinatorial optimization problem, known as the Inventory-Routing Problem (IRP) (4; 32). The

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IRP focuses on the determination of optimal replenishment plan and vehicles' routing schedule (11). Thus, it is applicable when the Vendor Managed Inventory (VMI) business model is followed (8). According to the VMI strategy, a supplier is responsible to manage and optimize the replenishment decisions of several customers (32).

This work addresses the IRP with strict driving hours' regulations and driving speed limits. More specifically, two variants of the problem under investigation are proposed, based on the adoption of different replenishment policies. The first problem case considers a strict replenishment policy, known as Just in Time (JiT), while the other adopts a Flexible Replenishment (FR) policy (15; 16). The main objective of this work is the examination of potential benefits of using a flexible replenishment policy under the consideration of practical operational restrictions.

2 Literature Review

Zachariadis et al. (2009) studied a single-depot IRP over a finite planning horizon for the service of multiple customers with constant deterministic demand for a single-type of product, using a fixed-fleet of vehicles (32). They proposed a flexible replenishment policy. Thus, the aim of the problem was the optimization of the overall supply chain network cost, through the simultaneous configuration of routes' schedule and the size and timing of replenishment. The proposed problem was formulated as a Mixed-Integer Linear Programming (MILP) model. Additionally, a hybrid Tabu Search-based heuristic solution method was developed for tackling efficiently large problem cases. Liu and Lee (2011) focused on the examination of a single-depot IRP with soft time windows (19). The goal of the problem was the determination of routing and inventory decisions to minimize the overall system cost, for the service of multiple customers with stochastic demands, under the consideration of a continuous review Economic Order Quantity (EOQ) policy. A Mixed-Integer Non-Linear Programming (MINLP) model and a two-phase hybrid Tabu Search - Variable Neighborhood Search heuristic solution method were developed by the authors to achieve high-quality solutions of small and large problem cases, respectively.

Coelho and Laporte (2014) focused on the improvement of IRP exact solutions by considering classic and new valid inequalities combined with input data ordering (11). They assumed a single-depot, multi-period, multi-vehicle IRP in which inventories held both in depot and customers' locations. Customers had deterministic demands and they were serviced by a capacitated fleet of homogeneous vehicles. The authors based on the relation between customers' demand and available capacities proposed three new valid inequalities. Moreover, they proposed input data ordering tactics. The first one ranks customers according to their total demand, in order to service first those with higher demand. The second one focuses on providing service priority to customers close to the location of supplier, while the third ordering tactic focuses on serving the most distant customers first.

Coelho et al. (2014) presented a review on the research contributions of IRP, by classifying them according to the problem variants, models and solutions approaches (9). Despite the lack of a standard problem version, the authors used the terms “basic versions” and “extensions of the basic versions”. The first category included IRP cases which commonly studied by research community up to 2012. The further classification of research contributions of basic problem versions conducted according to the following seven criteria:

- Time horizon.
- Structure.
- Routing.
- Inventory policy.
- Inventory decisions.
- Fleet composition.
- Fleet size.

Additionally, for the classification of research works on IRP extensions, the authors highlighted one more criterion, the “Products”, with three potential choices (single, two and multiple).

Mjirda et al. (2014) studied a multi-product IRP in which the objective goal was the simultaneous minimization of transportation and inventory costs (22). The problem was based on the service of an assembly plant, which faced deterministic demand for several products, by multiple suppliers and an unlimited fleet of capacitated vehicles over a finite planning horizon. The authors developed an MILP model as well as a Variable Neighborhood Search heuristic method with two phases for the solution of the addressed problem. A stochastic MILP was developed by Abdul Rahim et al. (2014) for the study of a multi-period stochastic IRP, in which a single depot services several retailers with homogeneous fleet of vehicles to satisfy their stochastic demand for a single type of product (1). The authors proposed a deterministic equivalent of their stochastic formulation and presented a Lagrangian relaxation method for the solution of medium-sized problem instances.

Coelho et al. (2014) focused on the development of Adaptive Large Neighborhood Search heuristics for the solution of dynamic and stochastic versions of IRP (8). They proposed two main frameworks, the proactive policy which utilizes demand forecast and the reactive policy, in which as (s,S) replenishment policy is adopted. Moreover, for each one of these policies, the authors investigated the impact of emergency lateral transshipment on the final solution quality. All policies were implemented in a rolling horizon fashion. Qin et al. (2014) investigated a periodic IRP with a FR policy for a single type of product (23). They assumed a two echelon supply chain system, in which a supplier has to satisfy periodic-variable demands of several retailers, using homogeneous capacitated vehicles. To solve realistic problem cases, the authors developed a Tabu Search-based algorithm.

A selective variant of the periodic IRP, applied on biodiesel production domain, was studied by Aksen et al. (2014) (2). More specifically, homogeneous capacitated vehicles depart from a biodiesel production facility and visits selected source nodes to collect waste vegetable oil according to the predetermined daily production plan of the production facility. The uncollected amount of waste oil is stored by source nodes until the next visit of a vehicle. To determine an optimized periodic collection schedule, the authors developed an MILP model. As the addressed problem is NP-hard, exact approaches are at least inefficient in solving problem cases of practical interest. Consequently, the authors adopted the Adaptive Large Neighborhood Search metaheuristic framework to develop an efficient heuristic solution method, which produced high quality solutions in short CPU times. A multi period IRP arisen from industrial gas distribution domain was studied by Singh et al. (2015) (26). The problem was formulated as an MILP model. To tackle large-scale problem cases, the authors proposed an incremental solution approach based on the decomposition of customers' set and the application of a randomized local search method for the solution of each subproblem.

Roldán et al. (2016) presented a survey of scientific literature on IRP with stochastic demands and lead times, focusing on multi-depot problem cases (24). They underlined the critical role of demands' variations and lead times' behavior on making high-quality decisions. Turan et al. (2016) studied a stochastic IRP which considers transshipments in specific time points in given time intervals for inventory rebalancing purposes (30). To tackle the high computational complexity, they designed and implemented a heuristic solution method, based on the VNS metaheuristic framework, combined with a dynamic programming operator. Archetti and Grazia Speranza (2016) investigated the potential benefit of integrating routing and inventory decisions (5). More specifically, they studied a separate version of the problem by following the Retailer Managed Inventory (RMI) policy and an integrated approach, which characterized by VMI policy. The problem variant under the RMI policy was solved in two stages, the delivery schedule and the route optimization stage, while the other one was tackled as a pure IRP. According to their findings, the authors highlighted that the potential benefits from integrated approach can be on average 9.5% and 9% in terms of total cost and number of selected vehicles respectively.

Li et al. (2016) focused on the study of characteristics such as the replenishment lead-times and inventory inaccuracy, which frequently met in fresh products' supply chains (18). The authors proposed a robust policy for the determination of routes and replenishment quantities. The proposed policy was divided into three steps. In the first one, the update of the current net inventory probability and the corresponding future forecast was performed. In the second step, a Robust Replenishment Time, Replenishment Quantity and Replenishment Stage Length algorithm was developed to optimally determine replenishment time, quantity and stage length, while in the third step the optimization of delivery routes was executed, by using a Genetic Algorithm. Soysal (2016) developed a probabilistic MILP model to formulate a closed-loop IRP, which considers forward and reverse logistics operations, fuel consumption and demand uncertainty for multiple products under the VMI policy (27). Moreover, a simulation model was developed to evaluate the solutions both of probabilistic model and its deterministic variant.

Schuijbroek et al. (2017) developed a cluster-first, route-second heuristic algorithm to solve an IRP based on bike sharing networks (25). Both MILP and constraint programming (CP) formulations are provided for the problem under investigation. The numerical experiments indicated that the proposed solution method outperformed the solutions of MILP and CP. A green variant of the IRP was studied by Cheng et al. (2017) under the consideration of environmental and economic issues and utilization of heterogeneous vehicles (7). The computation of fuel consumption was achieved by using the Comprehensive Modal Emission Model. According to this model, several characteristics can significantly affect the consumption of fuel in case of utilized vehicles. Such characteristics are the engine friction factor, the curb weight and the speed of the selected vehicle. Based on their findings, the authors underlined significant benefits achieved by using a mixed-fleet of vehicles.

Dong et al. (2017) developed novel solution methods for the solution of IRPs which consider several vehicle-based realistic constraints, such as variable consumption rates, time windows and driver constraints, following a VMI policy (12). The proposed solution method consists of three components. Initially, a reduction of search space is achieved by applying an elimination of redundant arcs. Next, a decomposition approach divides the problem into a VRP and a scheduling problem. After the solution of these two subproblems, an iterative approach of applying different integer cuts and parameter updates is performed to avoid potential infeasibilities or to address further cost improvements. Çankaya et al. (2018) studied an IRP for the distribution of humanitarian relief supplies to affected areas, known as the Inventory Slack Routing Problem (6). The objective goal in this problem case is the maximization of the minimum safety stock level at each affected location. To tackle the problem, the authors proposed a three-phase heuristic algorithm.

Malladi and Sowlati (2018) presented a comprehensive literature review of studies on Sustainable IRP (20). They focused on initially categorizing the selected research contributions into studies with single objective or multiple objectives. Further classification was performed based on the nature of objectives. More specifically, the studies with single objective categorized to those who focused on reverse logistics for waste collection, on closed-loop networks for the management of returnable items and on distribution networks of perishable products for the prevention of potential waste. The multi-objective works mainly focused on the combination of social and/or environmental objectives within the classic economic-related objectives. Wei et al. (2019) studied an extension of IRP, the cold-chain IRP which focuses on the distribution of food packages from a supplier to several customers (31). The customers close to depot are serviced by self-owned vehicles in short time limits, while the customers, who located far from the depot, are serviced by outsourced vehicles under long time limits. The problem was formulated as an MILP model and solved by a hybrid Genetic Algorithm with a descriptive model.

Soysal et al. (2019) presented a review on sustainable IRPs. They mainly focused on characteristics such as the model structure of objective and the solution approach (28). Based on sustainability concerns, they categorized the studies into those which tackled waste management products' perishability and emissions. Finally, the authors provided a detailed presentation over sustainable characteristics mentioned in the literature. Su et al. (2020) developed an MILP for a challenging IRP, faced by air-product companies and considers several realistic characteristics, such as heterogeneous fleet, time windows, multiple periods and heterogeneous drivers (29). To efficiently solve the problem, the authors proposed a matheuristic, which integrates exact methods applied to optimize timing and delivery quantities and a multi-neighborhood metaheuristic framework to schedule the routes.

Eide et al. (2020) studied a maritime IRP and focused on the optimization of vessels' speed considering their load to minimize fuel consumption and consequently sailing costs (13). As the dependency between fuel consumption and the factors of load and speed is not linear, the authors presented a linearization approach. Coelho et al. (2020) studied a multi-attribute IRP which extended the multi-depot IRP by further considering multiple products, heterogeneous fleet of vehicles and constraints relative to route duration (10). To efficiently solve this complex problem, the authors proposed a matheuristic method by combining exact methods with Variable Neighborhood Search.

Manousakis et al. (2021) investigated the IRP with Maximum Level inventory policy in a VMI system. They developed a novel two-commodity flow formulation and proposed six new valid inequalities (21). To solve classic hard problem instances, the authors proposed a Branch & Cut algorithm. Moreover, a combination of families of cuts and two separation strategies were also proposed to improve the performance of the developed algorithm. Alinaghian et al. (2021) focused on the investigation of an environmental-friendly IRP with time windows (3). They formulated the problem as an MILP model and focused on the simultaneous optimization of economic and environmental criteria. Moreover, the authors developed three heuristic solution methods based on Tabu Search and Differential Evolution metaheuristic frameworks. According to their numerical analysis, the augmented Tabu Search algorithm proved the most efficient solution method for the problem under invstigation.

3 Problem Statement & Mathematical Formulation

The IRP with strict driving hours' regulations and driving speed limits is examined over a finite planning horizon $T = \{1, ..., t\}$. A complete undirected graph $G = (V, E)$ is utilized to define the studied problem, where $V = \{1, ..., n, n + 1\}$ is the vertex set and $E = \{(i, j) : i, j \in V, i \neq j\}$ is the edge set. The main supplier of the supply chain system is denoted by the vertex $n + 1$, while the remaining vertices $I = V - \{n + 1\}$ represent n retailers. In the supplier location no inventory is held. However, it station a fleet of homogeneous capacitated vehicles, with a capacity level equals Q. Each retailer faces a period-variable deterministic demand for a single-type product, d_{it}. Moreover, a unit holding cost h_i is associated to each retailer. Focused on operational-related costs and restrictions, a routing cost c_{ij} for each pair of locations (i, j), specific driving speed limits, driving hours' regulations, UTL and drivers' wages, fd are considered.

To develop mathematical formulations of the proposed problems, the following variables are introduced:

- x_{ijklt}, a binary decision variable which equals 1, if vehicle k moves directly from node i to node j, with a speed level l, in time period t.
- q_{ikt}, a positive decision variable, which declares the quantity of product delivered to retailer i, using the vehicle k, in time period t.
- w_{itp}, a positive decision variable utilized in case of problem with flexible replenishment, which declares the delivered quantity of product to retailer i in the time period p to, wholly or partially, satisfy its demand in time period t.

Moreover, a positive variable, U_{ikt} $(0 \leq U_{ikt} \leq |I| - 1)$, is utilized to address the Miller–Tucker–Zemlin subtour elimination constraints.

The proposed mathematical models, of the under investigation problems, are modified extensions of the IRP model provided by Qi et al. (2014). Herein, a Mixed Integer Linear Programming (MILP) model of the IRP with strict driving hours' regulations and driving speed limits following the JiT replenishment policy is provided:

$$\begin{aligned} \min \sum_{i \in I} h_i \cdot \sum_{t \in H} \frac{1}{2} \cdot d_{it} \; + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{l \in L} \sum_{t \in T} c_{ij} \cdot x_{ijklt} \\ + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} \sum_{l \in L} \sum_{t \in T} fd \cdot \frac{(x_{ijklt} \cdot c_{ij})}{s_l} \end{aligned} \tag{1}$$

Subject to

$$\sum_{j \in V} \sum_{l \in L} x_{ijklt} - \sum_{j \in V} \sum_{l \in L} x_{jiklt} = 0, \qquad \forall\, i \in V, \forall\, k \in K, \; \forall\, t \in T \tag{2}$$

$$\sum_{j \in V} \sum_{k \in K} \sum_{l \in L} x_{ijklt} \leq 1, \qquad \forall\, i \in I, \; \forall\, \mathrm{t} \in \mathrm{T} \tag{3}$$

$$\sum_{j \in V} \sum_{k \in K} \sum_{l \in L} x_{jiklt} \leq 1, \qquad \forall\, i \in I, \; \forall\, \mathrm{t} \in \mathrm{T} \tag{4}$$

$$\sum_{i \in I} \sum_{l \in L} x_{i(n+1)klt} \leq 1, \qquad \forall\, k \in K, \; \forall\, \mathrm{t} \in \mathrm{T} \tag{5}$$

$$U_{ikt} - U_{jkt} + \left(|I| \cdot \sum_{l \in L} x_{ijkt} \right) \leq |I| - 1, \qquad \forall\, i, j \in I, \forall\, k \in K, \forall\, t \in T \tag{6}$$

$$\sum_{i \in I} q_{ikt} \leq Q_k, \qquad \forall\, k \in K, \forall\, t \in T \tag{7}$$

$$\sum_{k \in K} q_{ikt} = d_{it}, \qquad \forall\, i \in I, \forall\, t \in T \tag{8}$$

$$q_{ikt} \leq M \cdot \sum_{j \in V} \sum_{l \in L} x_{ijklt}, \qquad \forall\, i \in I, \forall\, t \in T, \forall\, k \in K \tag{9}$$

$$\sum_{j \in V} \sum_{l \in L} x_{ijklt} \leq M \cdot q_{ikt}, \qquad \forall\, i \in I, \forall\, k \in K, \forall\, t \in T \tag{10}$$

$$\sum_{l \in L, l > 3} x_{ijklt} = 0, \qquad \forall\, i, j \in I, \forall\, k \in K, \forall\, t \in T,\; c_{ij} \in [0, \frac{\overline{c}}{3}) \tag{11}$$

$$\sum_{l \in L, l > 8} x_{ijklt} = 0, \qquad \forall\, i, j \in I, \forall\, k \in K, \forall\, t \in T,\; c_{ij} \in [\frac{\overline{c}}{3}, \frac{\overline{c}}{2}] \tag{12}$$

$$\sum_{i \in I} \sum_{j \in I, j \neq i} \sum_{l \in L} \frac{c_{ij} \cdot x_{ijklt}}{s_l} < UTL, \qquad \forall\, k \in K, \forall\, t \in T \tag{13}$$

$$\frac{c_{ij} \cdot x_{ijklt}}{s_l} < UTL, \qquad \forall\, i, j \in I, \forall\, k \in K, \forall\, l \in L, \forall\, t \in T \tag{14}$$

The objective function of the model focuses on the optimization of the overall system cost, by minimizing the sum of all separate cost terms, such as the average inventory holding cost, the routing-related cost and drivers' wages. Constraints 2 impose the balance between inbound and outbound flow of vehicles in each node of the supply chain network. Constraints 3 and 4 guarantee that each retailer will be serviced by one vehicle at most in each time period. A vehicle can perform at most one route in each time according to Constraints 5. Constraints 6 secure the elimination of subtours for each utilized vehicle in each time period. Constraints 7 ensure the avoidance of any violation on the capacity of vehicles. Constraints 8 guarantee that the total delivered quantity of product to a retailer will be equal to the corresponding demand for the same time period. Constraints 9 and 10 impose that a delivery to a retailer will be addressed if and only if a visit will be scheduled. Constraints 11 and 12 address the compliance with driving speed limits, whilce Constraints 13 and 14 impose the driving hours' regulations. It is clear from the mathematical formulation of the problem case which adopts the strict JiT replenishment policy, that the problem is being degraded to a rich capacitated Vehicle Routing Problem (VRP) with consideration of average inventory holding costs.

Focusing on the IRP with strict driving hours' regulations and driving speed limits with FR, its MILP formulation is structured by considering Constraints 2 - 7 and 9 - 14 from the previous presented model plus the following new expressions:

$$\min \sum_{i\in I} h_i \sum_{t\in H}\left(\frac{1}{2}d_{it} + \sum_{p\in T, p<t} w_{itp}\,(t-p) + \sum_{p\in T, p>t} w_{itp}\,(t-p+|T|)\right)$$
$$+\sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T} c_{ij}\cdot x_{ijklt} \tag{15}$$
$$+\sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{l\in L}\sum_{t\in T} fd\cdot \frac{(x_{ijklt}\cdot c_{ij})}{s_l}$$

Subject to

$$\sum_{p\in H} w_{itp} = d_{it} \quad \forall i\in I,\ \forall \mathrm{t}\in \mathrm{H} \tag{16}$$

$$\sum_{t\in H} w_{itp} = \sum_{k\in K} q_{ikp} \quad \forall i\in I,\ \forall p\in H \tag{17}$$

$$w_{itp} \le d_{ip} \quad \forall i\in I,\ \forall t,p\in H \tag{18}$$

The objective function of this model considers the minimization of further inventory holding costs, such as the penalty costs produced due to potential deferred deliveries. The average inventory holding cost, the routing and the driver wages' costs remain in this formulation. According to the constraints, the main difference of this second formulation is the exclusion of Constraints 8 and the addition of three families of Constraints which address the flexibility of replenishment. More specifically, Constraints 16 impose that the product quantity delivered to a retailer in all time periods for satisfying his demand in a specific time period must be equal with that demand. Constraints 17 guarantee that the scheduled deferred deliveries to a retailer must be equal to the actual delivered quantities. Finally, the scheduled deferred deliveries cannot exceed the demand of a retailer for a specific time period, as it is addressed by Constraints 18.

4 Computational Results & Analysis

4.1 Computing Environment

The implementation of MILP models was made using the Gurobi Python API's, under an academic licence. The execution time limit for Gurobi Optimizer was set at two hours. The optimization solver ran on a laptop PC (windows 10 Home 64-bit), with an Intel Core i7-9750H CPU at 2.6 GHz and 16 GB RAM.

4.2 Problem Instances & Model Parameters

Ten small- and medium-sized problem instances were randomly selected by the works of Karakostas et al. (2019;2020), and they were slightly modified to fit to the problems under investigation (14; 17). More specifically, the first line of each problem instance provides the number of retailers, vehicles, and time periods. The second line provides depot-related data, such as its coordinates and its capacity. The following lines includes retailers' data, such as their coordinates, their holding costs and their period-variable demands. The last line provides the capacity of the homogeneous fleet of vehicles. The name of instances keep their current form A-B-C. "A" declares the number of potential depots (only one needed in this study), "B" is the number of retailers, and "C" represents the number of time periods. The drivers' wage was randomly generated in the interval $[12, 18]$, the big M parameter was arbitrarily set equal to the value $100 \cdot Q$, while the UTL was set at 8 hours following the EU regulations of working (`https://europa.eu/youreurope/business/human-resources/working-hours-holiday-leave/working-hours/index_en.htm`) and driving (`https://www.gov.uk/drivers-hours/eu-rules`) hours. The matching between speed levels and their corresponding values are provided in Table 1.

Table 1: The speed levels with their corresponding values

Speed level (l)	**Speed value** (km/h)
1	30
2	40
3	50
4	60
5	70
6	80
7	90
8	100
9	110
10	120

4.3 Numerical Results

Herein, the numerical results obtained by the Gurobi solver in case of the 10 random problem instances, are provided. Table 2 summarizes these results. Initially, the names of problem instances are given in the first column. Next, the second column presents the best found objective values for the IRP under the JiT policy and the third column provides the corresponding CPU times (s). The fourth column presents the best found objective values for the IRP under the FR policy, while the fifth one gives the CPU times (s), required by the solver to obtain those solutions. The asterisk symbol denotes the optimality of a solution.

Table 2: Total cost of best found solution of each instance.

Instance	TC_{JiT}	**CPU_time**$_{JiT}$	TC_{FR}	**CPU_time**$_{FR}$
4-8-3	1451.93*	9.99	1199.87	7200
4-8-5	2060,38*	11.57	1016.77	7200
4-10-3	1876.98*	808	1344.86	7200
4-10-5	2401.05	7200	1718.17	7200
4-12-5	2491.76*	2217.37	1572.11	7200
4-15-3	1647.73	7200	1187.24	7200
5-12-3	1863.07	7200	1310.72	7200
5-15-3	2130.13	7200	1685.18	7200
5-18-5	3022.8	7200	1973.39	7200
5-20-3	2477.7	7200	1886.89	7200
Average	2142.35	-	1489.52	-

According to the numerical results, using the FR policy leads to almost 30.5% better solutions in average. This significant decrease of overall cost is mainly achieved by the reduction of deliveries to the retailers. Figures 1, 2, 3 and 4 illustrate the quantities and the timing of replenishment, in case of two retailers from different problem instances, under the effect of JiT and FR. Moreover, these illustrations help the reader to understand the impact of FR policy on the reduction of frequent deliveries.

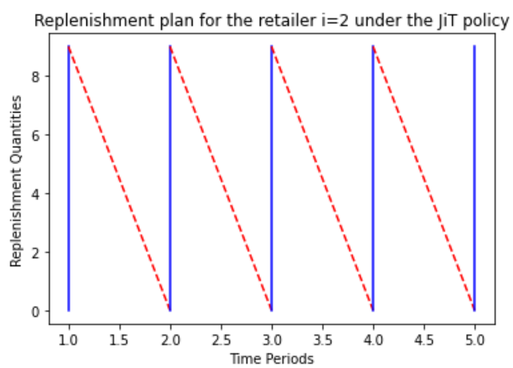

Fig. 1: An example of the replenishment plan for the retailer $i = 2$ according to JiT policy in instance $4 - 12 - 5$.

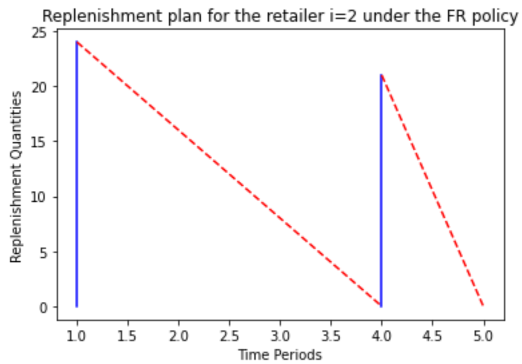

Fig. 2: An example of the replenishment plan for the retailer $i = 2$ according to FR policy in instance $4 - 12 - 5$.

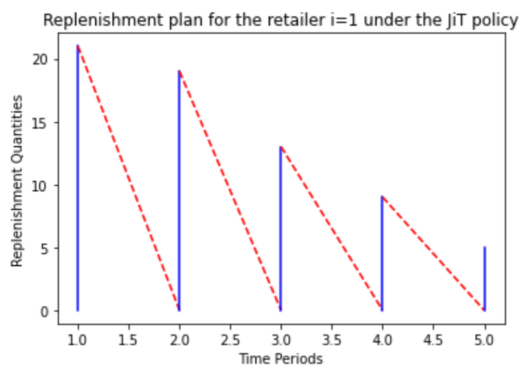

Fig. 3: An example of the replenishment plan for the retailer $i = 1$ according to JiT policy in instance $5 - 18 - 5$.

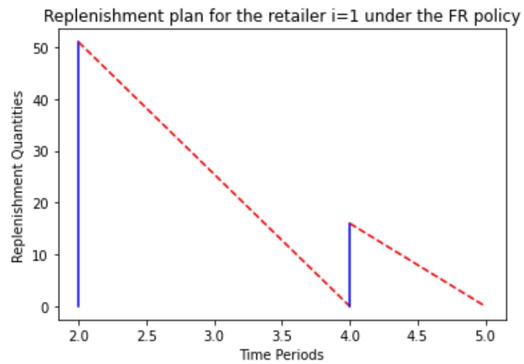

Fig. 4: An example of the replenishment plan for the retailer $i = 1$ according to FR policy in instance $5 - 18 - 5$.

Focusing on the structural characteristics of obtained solutions, the maximum number of required vehicles in the final solution of each model for each problem case is presented in Table 3.

Table 3: Maximum number of required vehicles.

Instance	**Jit**	**FR**
4-8-3	4	4
4-8-5	2	3
4-10-3	4	5
4-10-5	2	3
4-12-5	3	5
4-15-3	2	2
5-12-3	3	4
5-15-3	3	3
5-18-5	3	3
5-20-3	3	4

The adoption of FR policy leads to the utilization of more vehicles than following the Jit policy. However, focusing separately on the required vehicles in each time period, it is observed that following the FR policy leads to the use of less vehicles than those required under the adoption of JiT policy. An analytical presentation of vehicles' requirements per time period for each problem instance and for each replenishment policy is provided in Table 4, in the form $(x_1, ..., x_n)$ in case of n time periods. For example, the sequence $(2, 3, 3)$ denotes a three-time periods instance in which two vehicles are needed to perform the scheduled routes in the first period, and three vehicles in the second and the third time period respectively.

Table 4: Vehicles required per period under the adoption of each replenishment policy.

Instance	**Jit**	**FR**
4-8-3	(4, 4, 4)	(4, 4, 3)
4-8-5	(2, 2, 2, 2, 2)	(1, 2, 1, 1, 3)
4-10-3	(3, 4, 4)	(1, 4, 5)
4-10-5	(2, 2, 2, 2, 2)	(1, 3, 1, 2, 2)
4-12-5	(2, 2, 3, 2, 2)	(1, 5, 2, 1, 2)
4-15-3	(2, 2, 2)	(2, 2, 2)
5-12-3	(3, 3, 3)	(2, 1, 4)
5-15-3	(3, 3, 3)	(3, 3, 2)
5-18-5	(3, 3, 3, 3, 3)	(3, 3, 1, 2, 2)
5-20-3	(3, 3, 3)	(4, 2, 3)

This observation can highlight a further potential benefit by combining the FR policy with distribution outsourcing, which may significantly decrease the overall system cost, in case of fleet acquisition and usage costs' consideration.

4.4 Input Ordering

Herein, the potential impact of input ordering on the quality of final solutions is examined, by following the observations of Coelho and Laporte (2014) (11). More specifically, the first ordering strategy based on the distance of retailers from the supplier. According to this strategy, the data of the most distant retailers were loaded first. Similarly, the second input ordering strategy focuses on the total demand of retailers. The data of retailers who demanded more product quantity, were loaded first. The numerical results are summarized in Tables 5 and 6.

Table 5: Total cost of best found solution of each instance under the first input ordering strategy.

Instance	TC_{JiT}	**CPU_time**$_{JiT}$	TC_{FR}	**CPU_time**$_{FR}$
4-8-3	1451.93*	10.73	1204.16	7200
4-8-5	2060,38*	28.82	1016.77	7200
4-10-3	1876.98*	761.9	1324.5	7200
4-10-5	2401.05	7200	1693.58	7200
4-12-5	2491.76*	4291.67	1570.44	7200
4-15-3	1649.56	7200	1197	7200
5-12-3	1863.07	7200	1305.1	7200
5-15-3	2130.72	7200	1722.71	7200
5-18-5	3012.75	7200	1985.26	7200
5-20-3	2423.72	7200	1883.49	7200
Average	2136.19	-	1490.3	-

Table 6: Total cost of best found solution of each instance under the second input ordering strategy.

Instance	TC_{JiT}	CPU_time$_{JiT}$	TC_{FR}	CPU_time$_{FR}$
4-8-3	1451.93*	9.13	1198.23	7200
4-8-5	2060,38*	8.3	1016.77	7200
4-10-3	1876.98*	2068.73	1345.6	7200
4-10-5	2401.05	7200	1698.33	7200
4-12-5	2491.76*	7200	1561.32	7200
4-15-3	1648.01	7200	1183.74	7200
5-12-3	1863.07	7200	1305.73	7200
5-15-3	2130.13	7200	1761.45	7200
5-18-5	3023.49	7200	2072.64	7200
5-20-3	2477.41	7200	1896.25	7200
Average	2142.42	-	1504.01	-

The solutions produced by the Gurobi solver for the model using JiT replenishment policy, under the effect of the first input ordering strategy are slightly improved (0.3%), while those which obtained under the effect of the second input ordering strategy are almost equal to the initial ones. Similarly, no improvements have been achieved by following the first input ordering strategy for the case of FR policy. Moreover, the combination of FR within the second input ordering data leads to 1% worse solutions than the initial ones. Therefore, it can be highlighted that the input ordering is not an effective strategy to address further improvements for the studied IRPs.

4.5 Sensitivity Analysis

According to EU regulations of working (`https://europa.eu/youreurope/business/human-resources/working-hours-holiday-leave/working-hours/index_en.htm`) and driving (`https://www.gov.uk/drivers-hours/eu-rules`) hours, the main version of the models consider an 8-hour upper time limit of driving. However, focused only on the driving hours' regulations of EU, two other schedules are also permitted. The first one considers a driving regulation of nine hours, while the last one relax this limit up to 10 hours of driving. Therefore, a sensitivity analysis on the variations of driving upper time limit and its impact on final solution of MILP model adopting the FR policy was conducted and it is presented in this section. The results obtained by the Gurobi solver for each case of driving regulations are provided in Table 7.

Table 7: Sensitivity analysis on the variations of driving hours' upper limit.

	UTL = 9 hours		UTL = 10 hours	
Instance	TC_{FR}	**CPU_time**$_{FR}$	TC_{FR}	**CPU_time**$_{FR}$
4-8-3	1205.2	7200	1193.76	7200
4-8-5	1020.89	7200	1016.77	7200
4-10-3	1324.5	7200	1344.86	7200
4-10-5	1706.16	7200	1699.13	7200
4-12-5	1565.08	7200	1595.01	7200
4-15-3	1177.36	7200	1179.83	7200
5-12-3	1322.36	7200	1319.3	7200
5-15-3	1722.88	7200	1701.18	7200
5-18-5	2026.78	7200	2015.38	7200
5-20-3	1912.33	7200	1818.39	7200
Average	1498.35	-	1488.36	-

Despite UTL is a critical parameter of the proposed model, no significant changes were observed by the application of EU driving regulations' schedules (-0.59% and 0.08% respectively). This observation can be potentially pointed as a characteristic of robustness for the proposed MILP model.

5 Conclusion

This work presents two new Inventory Routing Problem variants by considering two replenishment policies and realistic operational constraints, such as driving speed limits and driving hours' regulations. The addressed problems were formulated as Mixed-Integer Linear Programming models and they were implemented using the interface of Python-Gurobi. To investigate the new problems, 10 Location IRP instances were randomly selected from literature and they were properly modified. Next, they were solved by the Gurobi solver. The produced solutions highlight the significant benefits by adopting a flexible replenishment policy under the Vendor Managed Inventory strategy. Moreover, an analysis on the effect of different input data ordering was conducted based on the findings noted in the literature (11). However, no significant changes were observed. Finally, a sensitivity analysis on the variations of driving upper time limit according to EU driving hours' regulations was performed.

The design and development of an efficient algorithm for the solution of more realistic problem cases is underlined as an interesting future research direction. Moreover, the consideration of further realistic characteristics, such as fuel consumption and service time in retailers will be an equivalently interesting future research.

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