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Mathematical optimization models for fuelwood production

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Abstract Forests are among the most sensitive systems in nature. This is attributed to the fact that, forests are directly affected by fluctuations in price of fossil fuels. Wood products and especially forest fuel products are accessible by anyone, without any prior processing. As forest fuel is a subsidy for fossil fuels (oil) for heating purposes, households turn to forest fuel especially in countries that are heavily impacted by economic recession. The over-exploitation of this natural resource leads the forest to abnormal situation and eventually to deforestation. The exhaust of the natural resource capital has negative impact not only on the local economy, where fuelwood market contributes especially in mountainous regions, but also on the environmental stability of ecosystems. In this paper, two multi-period Linear Programming (LP) models are proposed for management of coppice forests. The aim of these models is to maximize the Net Present Value (PV), which is constructed as a function of the revenue from trading fuelwood (price times the logged quantities) minus the transportation

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cost from the forest to merchants. Two aspects have been investigated in this paper; sustainability and maximum yield. The sustainability aspect is guaranteed by imposing constraints for equalization of non-logged areas at the end of the planning horizon. With maximum yield aspect, the maximization of the logged quantities (and therefore the maximization of the objective function) is guaranteed. The model is solved for various scenarios regarding transportation cost. The applicability of the model is demonstrated through a real-world case study of an even coppice forest in Achladochori-Aggistro-Sidirokastro. The proposed model is easy to be implemented, since it uses only the initial conditions of the forest (area) and can be applied to even and uneven aged forests.

Keywords Mathematical programming · Sensitivity analysis · Scheduling · Natural resources management

Mathematics Subject Classification (2000) 90C05 · 90C31 · 90B35 · 91B76

1 Introduction

Forest production and scheduling are among the most significant areas in conservation of natural resources. When logging is scheduled in a way so that environmental balance is guaranteed, the forest can potentially provide logging quantities theoretically for infinite time horizon. However, due to illegal logging, forests sustain heavy losses and environmental disorders each year, leading to decreasing wood quantities and eventually to deforestation. Recent economic recession constitutes a factor that affects the policy of natural resources and especially wood products (fuelwood). Illegal logging activity is enhanced by recession due to fact that household's income has decreased. Especially, in countries where the local population uses natural resources for heating purposes, illegal logging activity and ultimately deforestation is more likely to occur. The scheduling of a forest should not be implemented only based on environmental criteria but on financial as well since logging is a major part of the economic activity of mountainous places (where income from agriculture activities is not considerable). As logging activities should be implemented in a structured way, the question that rises is whether there is a way to optimally handle logged quantities so as to take into account the sustainability of the forest under both environmental and financial criteria. The proposed model is generic, provides valuable information, and can be implemented by any authority and relevant stakeholder. Based on the proposed Linear Programming (LP) forest scheduling model, policy makers can plan the logging period of coppice forest based on two aspects; environmental sustainability or maximum yield. In each of the two cases, aim of the model is the maximization of a financial function, which is constructed based on the logged quantities.

2 Literature review

Forest planning and scheduling is one of the most significant ways for planning and providing policy guidelines. Several works have been proposed in the world literature for modeling decision support systems regarding the planning and control of the production process (at the forest) or at distribution, energy production and wood products level. Regarding the production process a selection of mathematical programming techniques like Linear Programming (LP), Mixed Integer Linear Programming (MILP) or Multi-Objective Linear Programming (MOLP) models have been deployed [6,15,22]. Also, the use of each technique is depending on the type of the wood product of the forest. For instance the modeling of an oak type forest will be modeled entirely different than a eucalyptus or a coppice forest [7]. Salehi and Eriksson in their work [26], proposed an LP model for a mixed coppice oak forest. The area of application was southern Zagros – Iran; decision levels were provided on a 100 year time horizon for wood-fuel volume and harvest area. The modeling of Eucalyptus forests have also been proposed with combination of LP and MOLP models [7, 14]; the LP model is introduced to maximize timber harvests and Net Present Value (NPV) based on a Model II type forest [18]. Under the assumption of variations in land productivity the MOLP (Goal Programming) models are introduced to provide more stable results. The strategic plans under the presence of multiple stakeholders have also been examined using MOLP models [13]. Besides NPV maximization, the investigation of Land Expectation Value (LEV) has been also used as an LP objective function in forest coppice production – management models [5]. The study of even-aged forests has been firstly examined by Nautiyal and Pearse in [23]; thereafter even and uneven aged forests have been extensively investigated [1,12,29]. Besides LP models, forest dynamics models have been proposed to investigate un-even aged forests [8]. Flisberg et al. [10] have proposed an LP model in order to determine the optimal transportation of forest fuel, minimizing cost. This logistics optimization model aims at minimizing cost by taking into account operational and inventory constraints. Termansen [27] proposed a forest harvesting model based on Faustmann rotation model. Modern matheuristic methods combining simulated annealing and linear programming, have been applied to an old forest, in a long-term forest planning problem in order to derive effective solutions in a multi-objective programming model [24].

Apart of the models that handle forest production providing policy for planning of the area that will be harvested or will be logged, as mentioned before, there are also models that examine the way that wood products will be distributed to the next links of this supply chain. The forest products can be used as biomass for electrical or thermal energy production [11], for household use [2] or for producing technical wood and paper [4]. Besides forest supply chain, biomass supply chains have also been examined extensively in the world literature. Biomass may consist of several components such as industrial agricultural products (industrial corn) and forest residues. A selection of mathematical programming models and simulation techniques has been proposed in order to model the biomass supply chain and provide energy planning (throughout biofuel production) while reducing carbon emissions [30]. By using optimization models, it is possible to provide either tactical (day by day) decisions or long range decisions. Optimal design of supply chain networks is a modeling type which falls in the aforementioned category [25]. Regarding the design of supply biomass to biofinery supply chain, MILP models are usually applied to illustrate the logistics, warehousing, and to design the network [9, 19, 20]. An algorithm which is based on Data Envelopment Analysis (DEA) applied on biomass supply chain, has been proposed by Grigoroudis et al. in [16].

The proposed scheduling can be applied to even and un-even aged coppice forests providing generic modeling. This decision support tool can be used by experts for planning over a long time period regarding harvesting and logging of forest areas. Such decision support tools have been widely adopted by researchers for a plethora of natural resources models [28]. The objective of the proposed tool is to maximize PV while retaining the area that is not logged in acceptable levels. The proposed model extends previously proposed models and it comprises of enhanced characteristics for providing sensitivity analysis, while a real-world case study is shown to demonstrate its applicability.

3 Mathematical formulation

3.1 Nomenclature

Index

c: Age class $c = 1, \ldots, u$.
t: Time period $t = 1, \ldots, u$.
j: Scenarios $j = 1, \ldots, SC$.
d: Distance $d = 1, \ldots, D$.
i: Price $i = 1, \ldots, I$.
j: Return $j = 1, \ldots, J$.

Continuous variables

PV: Present value (€).
$F_{c,t}$: Initial area of class age c at time period t (hectares or simply ha).
$l_{c,t}$: Logged area of class age c at time period t (ha).

Parameters

u: Rotation age (years).
β: Percentage placed on bounds (%).
γ: Conversion factor to spatial cubic meters.
r: Interest rate (%).
p: Fuelwood unit price (€/ spatial cubic meters or simply sp.m^3).
F_c^0: Initial area of age class c for the first time period (ha).

$F^U_{c,t=u}$: Upper bound of initial area of age class c at the end of the planning horizon $(t = u)$ (ha).
$F^L_{c,t=u}$: Lower bound of initial area of age class c at the end of the planning horizon $(t = u)$ (ha).
V_c: Volume of fuelwood of age class c per logged area (m^3/ha).
$c\,(D)$: Transportation cost as a function of distance (€/km).
F^U: Upper bound of the initial area at the end of the planning horizon (m^3/ha).
F^L: Lower bound of the initial area at the end of the planning horizon (m^3/ha).

3.2 Mathematical formulation of LP scheduling models

In this section the mathematical formulation of the proposed multi-period LP scheduling approaches will be described. The proposed models extend the work of Galatsidas et al. [12] and examine the two states that a coppice forest can be transited; sustainability and maximum yield. In the first state, the scheduling model of the coppice forest aims to equal areas at the end of the planning horizon, while in the second state the model is designed in order to yield the maximum return in terms of growing stock and therefore in terms of monetary units. Both proposed models aim to maximize the Present Value (PV), which is defined as follows:

$$\max PV = \frac{\sum_c \sum_t V_c \cdot l_{c,t} \cdot \gamma \cdot (p - c(D))}{(1+r)^{t-1}} \tag{1}$$

In objective function (1), p stands for the unit price of fuelwood produced and is measured in $€/m^3$, while γ is a conversion factor that corresponds cubic meters (m^3) to spatial cubic meters (sp.m^3). Parameter V_c is introduced to convert the area logged to the corresponding volume of fuelwood based on [21]. As the objective function represents PV, r is the interest rate discounted at time period t and is set to 3%. Each time period corresponds to 5 years of planning (e.g., $t = 1$ corresponds to $1 - 5$ years, $t = 2$ to $6 - 10$ years, etc.). The values of parameter V_c are provided in Table 1. The conversion factor to spatial cubic meters is 0.67 while the unit price of fuelwood is 30 €/sp.m^3. In this paper, data for real forest conditions are provided. Although, the rotation age of the specific forest is $u = 20$ years, an additional dummy class age is created. This makes the scheduling of the LP model more easy, while provides also information regarding the area at the end of the planning horizon.

The area that will be left after the implementation of the scheduling model ($F_{c,t}$) should be greater than the logged area ($l_{c,t}$). The dimensions of the aforementioned continuous variables are $c = 1, \ldots, 5$ for the class age and $t = 1, \ldots, 5$. Based on this proposition for each class age and time period, the next constraint is introduced:

$$F_{c,t} \geq l_{c,t} \ , c \geq 3, t \leq 4 \tag{2}$$

Table 1 Values for fuelwood volume per age class.

Age class c / years	V_c (m^3/ha)
class 1 (1 – 5)	0
class 2 (6 – 10)	0
class 3 (11 – 15)	60
class 4 (16 – 20)	89
class 5 (20 – 25)	117

In the first period of the planning, the initial conditions are provided. The starting values for the initial are presented in Table 2. Thus, the next constraint is introduced:

$$F_{c,t=1} = F_c^0 \ , \ \forall c \tag{3}$$

Table 2 Values for initial area for the first time period of age class c.

Age class c / years	(ha)
class 1 (1 – 5)	0
class 2 (6 – 10)	90
class 3 (11 – 15)	780
class 4 (16 – 20)	959
class 5 (20 – 25)	886

The area that corresponds to the initial area, is transferred from the logged area of the previous time period. Thus, for the first class age of the initial area, the next constraint is introduced:

$$F_{c=1,t} = \sum_c l_{c,t-1} \ , t > 2 \tag{4}$$

After transferring the area to the first class age, the initial area for any class age $c \geq 2$ up to class age 4 (in this case) or up to the penultimate class age (in the general case) is computed. This initial area is defined as the subtraction of the logged area from the initial area, corresponding to the previous age class / time period, as modeled in the next constraint:

$$F_{c,t} = F_{c-1,t-1} - l_{c-1,t-1}, \ \forall c, \ t \geq 2 \tag{5}$$

From the proposed planning scheme, if over mature stands exists (stands that belong to the last class age 5), the initial area is calculated with the following constraint:

$$F_{c=5,t} = F_{c=4,t} - l_{c=4,t} + F_{c=4,t-1} - l_{c=4,t-1} \ \forall t \geq 2 \tag{6}$$

In order to prevent logging in the first age classes $(1, 2)$, where there are very young trees, the next constraint is introduced:

$$l_{c,t} = 0,\ \forall t,\ c < 3 \tag{7}$$

The initial area has a dummy age class, where no values should be assigned to. For that reason, the next constraint is introduced:

$$F_{c=5,t=5} = 0 \tag{8}$$

The variables of the study express initial and logged area and therefore cannot take any negative values. Thus, the next set of non-negativity constraints are imposed on the variables:

$$\begin{cases} F_{c,t} \geq 0 \\ l_{c,t} \geq 0 \end{cases} \forall c, t \tag{9}$$

At the end of the planning horizon, the Decision Maker (DM) (e.g., any governmental authority) according to his/her priorities and/or external factors (such as fires, natural disasters) or demand in fuelwood, can choose between sustainability or maximum yield as presented in the next sections.

3.2.1 Sustainability LP model

At the end of the planning horizon ($t = 5$), sustainability can be achieved by equal areas in age classes 1 – 4, since based on constraint (9) the initial area for age class 5 and time period 5 is 0, based on [12].

$$F_{c,t=5} = F_{c-1,t=5},\ c \leq 4 \tag{10}$$

This type of constraint is very hard to achieve and infeasibilities may occur. Thus, the next constraints are introduced, in order to provide more degrees of freedom to the LP model:

$$F_{c,t=5} \leq F^U,\ c \leq 4 \tag{11}$$

$$F_{c,t=5} \geq F^L,\ c \leq 4 \tag{12}$$

In the above inequalities (11) and (12), F^U and F^L are the upper and lower bounds, respectively, for the initial area at the end of the planning horizon. The upper and lower bounds on the initial area are defined as the total initial area divided into four age classes ($1 \leq c \leq 4$). In each instance, a percentage for differentiation is set (primarily to 1%).

$$F^U = (1 + \beta) \cdot \frac{\sum_c F_c^0}{4} \tag{13}$$

$$F^L = (1 - \beta) \cdot \frac{\sum_c F_c^0}{4} \tag{14}$$

Thus, in this case the total initial area equals to 2715 ha, by summing the rows of Table 2, and upper and lower bound are computed as follows:

$$\left[F_c^L,\ F_c^U\right] = [671.96,\ 685.53] \tag{15}$$

The sustainable LP model is formulated with objective function (1) and constraints (2)-(14).

3.2.2 Maximum yield LP model

In this case, the model provides maximum yield and therefore maximum PV. Based on maximum yield orientation towards the forest management, the initial area for certain ages classes equal to the logged area, as shown in the next constraint.

$$F_{c,t} = l_{c,t}\ ,\ t \geq 2,\ c \geq 4 \tag{16}$$

By adding constraint (17), variable $l_{c,t}$ is forced to load more area in age classes 4 and 5 for any time period $t \geq 2$, since this age class corresponds to the second largest objective coefficient, based on Table 1 and objective function (1).

In order to achieve maximum yield in age class 4, the initial area is defined as follows:

$$F_{c=4,t} = F_{c=3,t-1} - l_{c=3,t-1} + F_{c=4,t-1} - l_{c=4,t-1} + F_{c=5,t-1} - l_{c=5,t-1},\ t \geq 2 \tag{17}$$

Based on constraint (17), the initial area is defined from the subtraction of the previous time period age classes, and therefore no area moves to the last age class (class $c = 5$).

The maximum yield LP model is formulated with objective function (1) and constraints (2) – (7), (9), and (16), (17).

4 Results

This section analytically presents the results of the study. Based on real data [12] obtained from the forest management plan of Achladochori-Aggistro-Sidirokastro forest regarding the initial conditions, the results of the two scheduling LP models are shown. Especially for the sustainable LP model, a sensitivity analysis is implemented regarding the range of the bounds on the initial area at the end of the planning horizon as shown in constraints (12) – (15).

Both LP problems models have been modeled using the General Algebraic Modeling System (GAMS) [3]. They were optimally solved using the IBM ILOG CPLEX v. 12.7.1 solver [17].

4.1 Sustainability model

The results of the sustainable LP model are presented in Table 3 and correspond to LP model with percentage $\beta = 0$, thus making constraints (12) and (13) equivalent to (11). As it can be seen in Table 3, the initial conditions are shown in the first time period of planning. At the end of the planning horizon ($t = 5$), the initial area comes into sustainability as the area equals to 678.75 *ha* based on Table 3 (depicted with gray color). According to Table 4, results regarding the logged area is conducted for age stands with class greater than or equal to three. This is actually imposed by constraint (7). The advantage in this type of modeling is to "push" the logging area to larger age class stands (and therefore more mature) in order to maximize the total yield and therefore PV.

Table 3 Initial area results from sustainable LP model.

c / t	1	2	3	4	5
1	0	1357.5	678.75	678.75	678.75
2	90	0	1357.5	678.75	678.75
3	780	90	0	1357.5	678.75
4	959	780	90	0	678.75
5	886	487.5	588.75	0	0

Table 4 Logged area results from sustainable LP model.

c / t	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	0	0	0	678.75
4	471.5	191.25	90	0
5	886	487.5	588.75	0

4.1.1 Sensitivity analysis on the bounds

Various scenarios are studied, starting from $\beta = 1\%$, as a sensitivity analysis for the initial area. As it can be seen in Figure 1, by changing the percentage from 1% to 9%, the bounds change as well, (11) and (12), respectively. It can be also seen that for no change (0%), sustainability is achieved among age classes $1 - 4$. Changing the percentage (β), optimal values for the initial area change as well leading to unequal initial area between classes $1, 2$ and $3, 4$.

Based on Figure 2, the initial area is examined for the variables at the end of the planning horizon. According to Figure 2, the initial area for $\beta = 0\%$ was $F_{c=1,t=1} = F_{c=2,t=5} = F_{c=3,t=5} = F_{c=4,t=5} = 678.75$ *ha*, while $F_{c=5,t=5}$

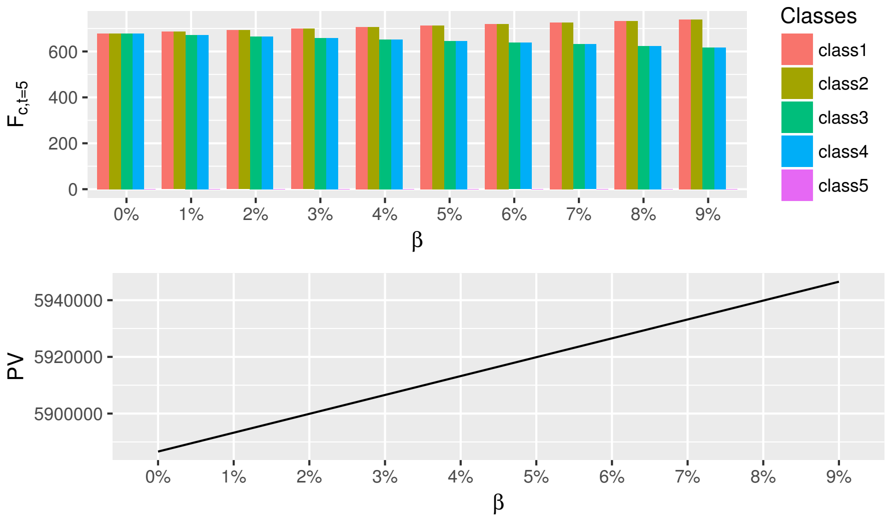

Fig. 1 Upper and lower bounds of constraints (12), (13), initial area at the end of the planning horizon, and PV for different percentages (β).

is zero. By performing the sensitivity analysis it can be seen that, the larger the difference between the upper and lower bound, the biggest the difference between the first two age classes ($c = 1,\ 2$) and age classes 3 and 4 (Figure 2). Furthermore, by expanding the upper and lower bounds on the initial area, the first two age classes tend to follow the upper bound while age classes 3 and 4 tend to follow the lower bound (Figure 1).

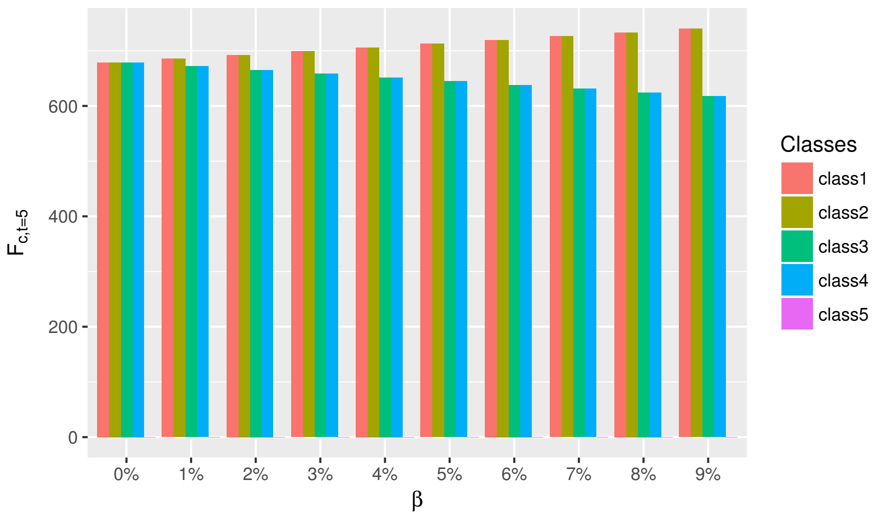

Fig. 2 Initial area at the end of planning horizon (t=5) for different percentages (β).

Changing the upper and lower bounds of constraints (11) and (12) cause the initial area at the end of the planning horizon ($F_{c,t=5}$) to receive unequal values among classes. Thus, in order to assess the differentiation of the optimal values, we compute the variance.

$$\sigma^2 = \frac{1}{|SC| - 1} \sum_{c} \left(F_{c,t=5} - \bar{F}_{c,t=5} \right)^2 \tag{18}$$

The results of the standard deviation are plotted compared to the values of the objective function PV, as it is depicted in Figure 3. Results indicate that as bounds increase, the variance of the values of the initial area at the end of the planning horizon ($F_{c,t=5}$) tend to increase and PV increases as well, as it is demonstrated in Figure 3.

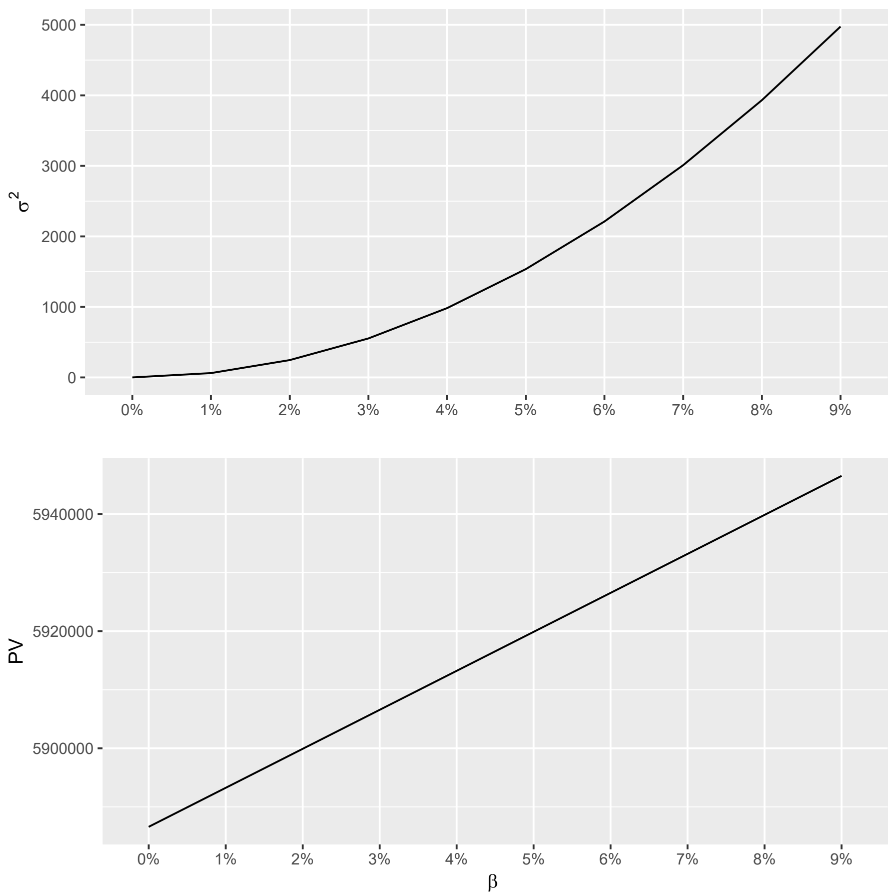

Fig. 3 Variance (σ^2) of the initial area ($F_{c,t=5}$) and PV at the end of the planning horizon for different percentages (β).

4.1.2 Sensitivity analysis for transportation cost

In this section a sensitivity analysis will be performed, regarding the values of PV for different types of cost based on the distance according to objective function (1). The scenarios of transportation cost are presented in Table 5, thus the LP model for sustainability is solved for nine scenarios regarding the bounds placed at constraints (11) and (12).

Table 5 Scenarios for transportation cost.

Transportation scenario	Cost/distance (€/sp.m^3 per 100 meters)
1	10.87
2	11.92
3	12.97
4	14.02
5	15.07
6	16.04
7	16.77
8	17.50
9	18.23
10	18.96

Results regarding the value of PV for each value of moving cost and percentage change in the bounds are also demonstrated in Table 6. As it can be seen in Table 6, as the moving cost increases, the PV value decreases but as the bounds on constraints (11) and (12) become wider, PV value slightly increases.

Table 6 Results for PV (€/) for each case of bound and transportation cost.

c(d) / β	1%	2%	3%	4%	5%	6%	7%	8%	9%
1	3,237,960	3,241,622	3,245,284	3,248,946	3,252,608	3,256,270	3,259,932	3,263,594	3,267,256
2	3,060,236	3,063,697	3,067,158	3,070,619	3,074,080	3,077,541	3,081,002	3,084,463	3,087,924
3	2,882,512	2,885,772	2,889,032	2,892,292	2,895,552	2,898,812	2,902,072	2,905,332	2,908,592
4	2,704,788	2,707,847	2,710,906	2,713,965	2,717,024	2,720,083	2,723,142	2,726,201	2,729,260
5	2,527,064	2,529,922	2,532,780	2,535,638	2,538,496	2,541,354	2,544,212	2,547,070	2,549,928
6	2,362,881	2,365,554	2,368,226	2,370,898	2,373,571	2,376,243	2,378,915	2,381,588	2,384,260
7	2,239,321	2,241,853	2,244,386	2,246,919	2,249,451	2,251,984	2,254,516	2,257,049	2,259,582
8	2,115,760	2,118,153	2,120,546	2,122,939	2,125,332	2,127,725	2,130,118	2,132,510	2,134,903
9	1,992,200	1,994,453	1,996,706	1,998,959	2,001,213	2,003,466	2,005,719	2,007,972	2,010,225
10	1,868,640	1,870,753	1,872,866	1,874,980	1,877,093	1,879,206	1,881,320	1,883,433	1,885,547

4.2 Maximum yield model

Similar to the sustainable LP model, the results for the initial area and logged maximum yield model are presented in Tables 7 and 8. As it can be seen, the area is "loaded" in the first two age classes at the end of the planning horizon. The results show very rough treatment on the initial area results at the end of the planning horizon (t=5), since from time periods 1 and 2 (c=1,2) the initial area that is loaded is $F_{c=1,t=5} = 2625$ and $F_{c=2,t=5} = 90\ ha$, respectively (Table 5).

Table 7 Initial area results from maximum yield LP model.

c / t	1	2	3	4	5
1	0	2625	0	90	2625
2	90	0	2625	0	90
3	780	90	0	2625	0
4	959	0	90	0	0
5	886	0	0	0	0

Table 8 shows that, the logged area is concentrated for age classes larger than 3; thus providing more wood yield than the sustainability model.

Table 8 Logged area results from maximum yield LP model.

c / t	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	780	0	0	2625
4	959	0	90	0
5	886	0	0	0

4.2.1 Sensitivity analysis on financial parameters

In this section sensitivity analysis will be conducted for the financial parameters, namely price (p), interest rate (r) and transportation cost ($c(D)$). For each set of parameters, the values that are examined are demonstrated in Table 9. Here 5 scenarios are examined for price and interest rate parameters ($I = J = 5$). Scenario 3 represents the current situation. The scenarios will be examined using the sustainability LP model with objective function (19).

$$\max PV^{i,j,d} = \frac{\sum_c \sum_t V_c \cdot l_{c,t} \cdot \gamma \cdot \left(p^i - c(d)\right)}{\left(1 + r^j\right)^{t-1}}, \ d = 1, \dots D, \quad i = 1, \dots I, \ j = 1, \dots J \qquad (19)$$

Table 9 Price and interest rate scenarios.

Scenario	Price (p^i) (€/sp.m^3)	Interest rate (r^j) (%)
1	20	0.02
2	25	0.03
3	30	0.04
4	35	0.05
5	40	0.06

The sustainability LP model that is solved for $|I| \times |J| \times |SC|$, whereas $|I| = 5$, $|J| = 5$ and $|SC| = 10$. The resulting LP model, which consists of objective function (19) and constraints (2)-(9) and (11), (12), is solved for 250 instances. In order to reduce the dimension of the scenarios and corresponding results, for the sustainability LP model, constraint (10) is introduced instead of constraints (11) and (12).

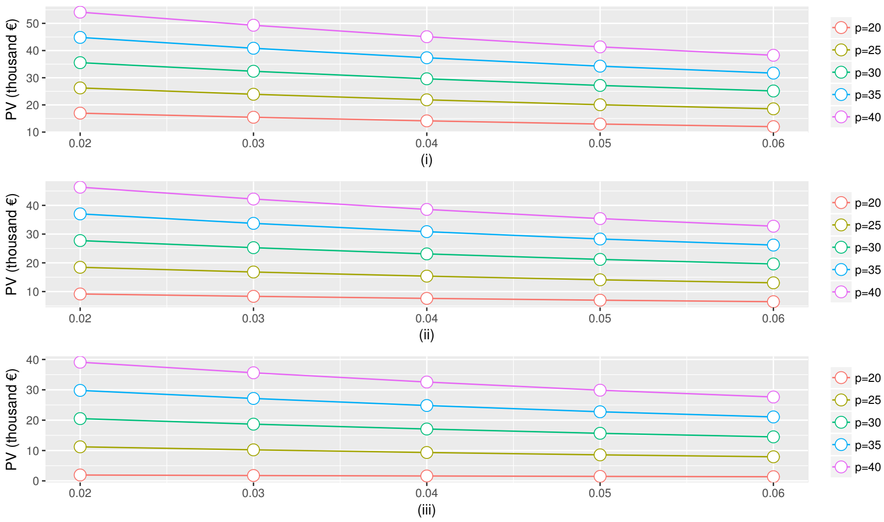

Fig. 4 Line plots of price (p^i) and interest rate (r^j) scenario examined for Present Value i) $PV^{i,j,d=1}$, ii) $PV^{i,j,d=5}$, and iii) $PV^{i,j,d=10}$.

The results of the objective function are presented in Figure 4. Each figure represents a line plot for the values of the objective function as presented in (19). The scenarios that are examined concern prices, interest rates percentages and transportation cost combinations. The latter scenarios are demonstrated in Table 9 and Table 5 respectively. However, in Figure 4, only three scenarios are examined regarding transportation cost; $c(1), c(5)$ and $c(10)$. Each of the lines represent PV for each of the examined scenario. Present Value curves tend to move downwards, leading to the assumption that PV receives lower values, when larger cost values are introduced. For example in Figure 4,i) the magenta color line (corresponding to 40€) is higher than the corresponding curve represented in Figure 4,ii) and Figure 4,iii). From Figure 4 it is concluded that the higher the values of the interest rate the lower the PV. general view of the PV is reflected in Figures 5 and 6. The latter figures present contour plots of the PV derived when LP model is solved for price and transportation cost scenario at interest rate values 2% and 6%. More specifically, in Figure 5 the curve corresponding to the highest value is approximately more than 50 thousand €and it is reported for prices in the range of $35-40$ € and $d \leq 2$. In the same range, the highest reported curve is reported for $PV \simeq 35$ thousand €.

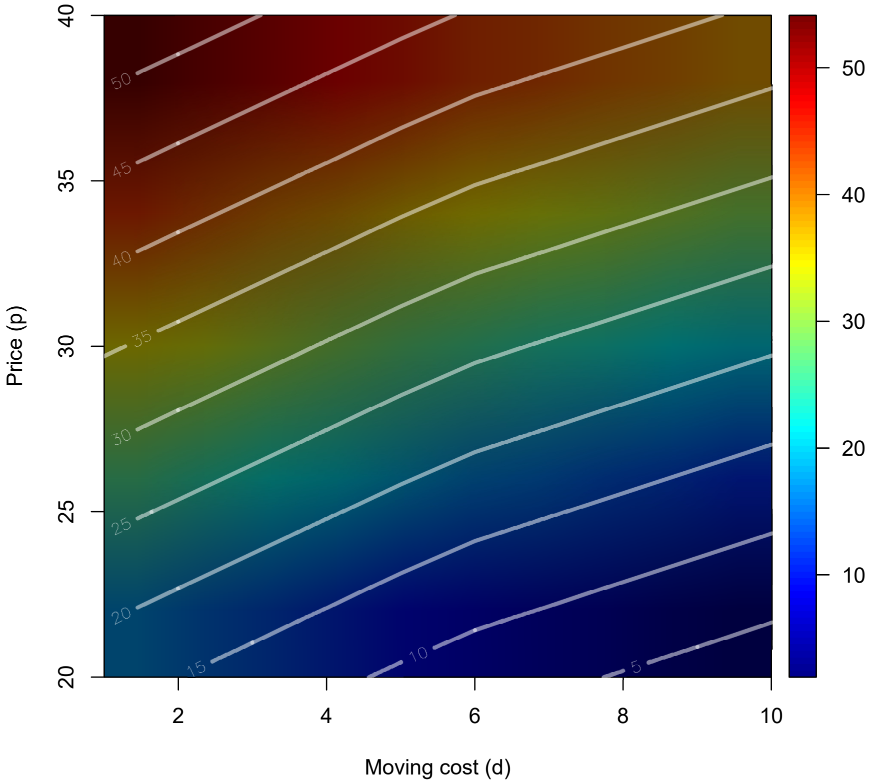

Fig. 5 Two dimensional image of Present Value (in thousand €) for price and transportation cost scenario at $r = 0.02$ ($PV^{d,i,j=1}$).

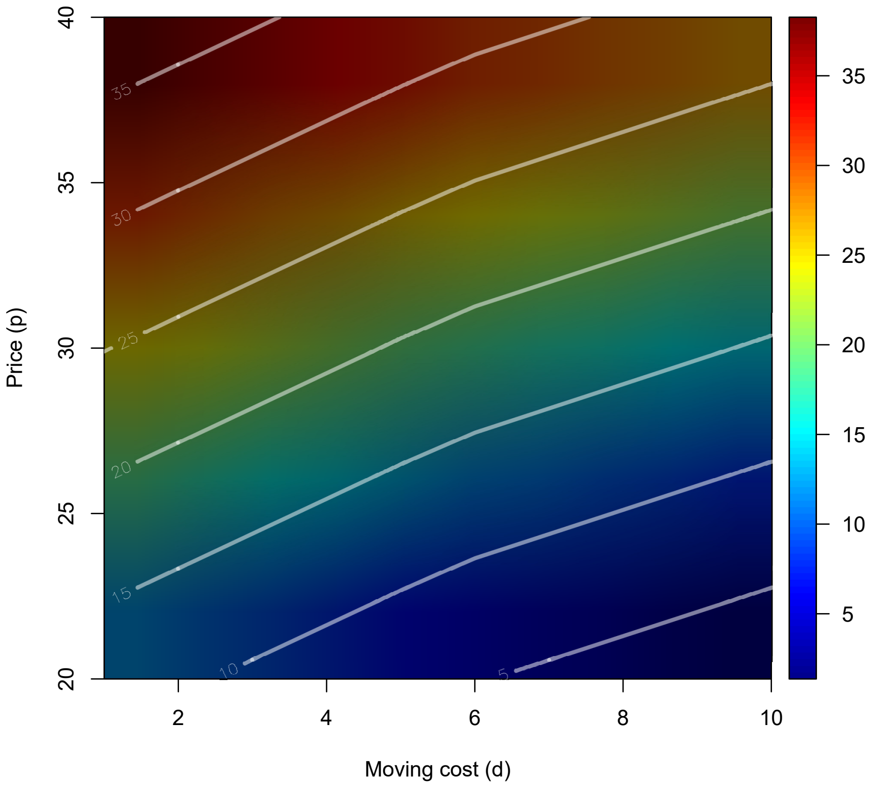

Fig. 6 Two dimensional image of Present Value (in thousand €) for price and transportation cost scenario at $r = 0.06$ ($PV^{d,i,j=5}$).

5 Conclusions

The scheduling of forest production is of major importance and ecological sustainability can be achieved by applying the best practices. Forest products are characterized as a renewable energy source under the condition of logging with certain rules. Otherwise, the source will not be renewable and will lead to the condition of deforestation and ecological instability. Especially for Greece, fuelwood is one of the most popular fuels for household heating, after the imposition of tax to oil and the decrease of household's income.

In the present work, two LP models have been investigated both aiming to the maximization of the PV under constraints for logged and initial area. The first model aims to the sustainability at the end of the planning horizon (age rotation). Sustainability is achieved when results show equalization of initial areas at the end of the planning horizon (age rotation). The contribution of this paper is that, the bounds placed at the end of the planning horizon allow the LP model to be parametrically solved, thus making it solvable for various values of initial area. The first LP model that targets at sustainability was solved for bounds in the range of ± 1 − 9% of 678.75 ha, which corresponds to the total initial area for $t = 1$ divided by the number of classes that sustainability is aim to ($\sum_c F_{c,t=1}/4$).

The second LP model that is examined in this paper aims to the maximization of the PV, without placing any constraint about the equalization of the initial area at the end of the planning horizon. Based on this approach, the logged area is larger on age classes larger than or equal to 3, where the objective coefficients are larger as well. The initial area scheme looks rougher than the sustainability model, since the initial area of the latter is gradually changing; leading to equal area at the end of the planning horizon. Results showed that, the objective of the second LP model (maximum yield) was significantly larger than the objective of the first LP model (sustainability).

A sensitivity analysis of the financial parameters has been conducted. For sustainability LP model, several scenarios have been examined regarding wood prices (p), transportation cost (c(D)) and rate of return (r). The new objective function that is constructed is denoted in equation (19). From the results it can be seen that PV curves decrease when transportation cost values (c(D)) and rate of return (r) parameters increase. These findings imply that more effective and low cost methods (e.g., better scheduling, bulk transportation) should be used for reducing cost, which subsequently can increase financial benefit.

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