

## Integer & mixed-integer programming

Mon.3.MA 042

### Linear optimization

Chair Angelo Sifaleras, University of Macedonia

Sergei Chubanov, University of Siegen

### An improved polynomial relaxation-type algorithm for linear programming

To find a solution of a system of linear inequalities, the classical relaxation method projects the current point, at every iteration, onto a hyperplane defined by a violated constraint. The constructed sequence converges to a feasible solution. It is well known that the method is not polynomial. One of the reasons for this is that each iteration considers only one violated constraint among the original constraints of the system. Unlike the relaxation method, each iteration of our algorithm considers an appropriate nonnegative linear combination of the inequalities. The algorithm runs in  $O(n^3 L_{\min})$  time where  $n$  is the number of variables and  $L_{\min}$  is the minimum binary size of a feasible solution. In particular, the algorithm either finds a nonnegative solution of a system of linear equations or proves that there are no 0, 1-solutions in  $O(n^4)$  time. This theoretical estimate is less by the factor of  $n^2$  than that of our previous algorithm.

Roland Wunderling, IBM

### The kernel simplex method

The Simplex Method has stopped seeing major computational advances for years, yet it remains the most widely used algorithm for solving LPs; in particular the dual Simplex algorithm is used for MIP because of its warm-start capabilities. State-of-the-art MIP solvers use branch-and-cut algorithms, but the standard dual simplex algorithm only addresses the branching aspect of it. When cuts are added usually a fresh factorization of the basis matrix is needed which greatly reduces true warm-start support. Using a row basis or dualization can mitigate the issue, but this is only efficient for models with more rows than columns.

In this talk we introduce a new simplex algorithm, the kernel simplex method (KSM), which defines a kernel instead of a basis as the central data structure. KSM, provides full warm-starting functionality for row and column additions or deletions. We describe the algorithm and differentiate its computational properties against the traditional simplex method. Further, we show how KSM unifies primal and dual algorithms into one symmetric algorithm, thus matching duality theory much better than the traditional methods.

Angelo Sifaleras, University of Macedonia (with Nikolaos Samaras)

### Exterior point simplex-type algorithms for linear and network optimization problems

The linear problem is one of the most useful and well-studied optimization problems, which is widely used in several areas of science. Lots of real world problems can be formulated as linear programs. The popularity of linear programming can be attributed to many factors such as the ability to model large problems, and the ability to solve large problems in a reasonable amount of time. Many algorithms have been invented for the solution of a linear program. The majority of these algorithms belong to two main categories: (i) Simplex-type or pivoting algorithms and (ii) interior-point methods (IPMs). All the algorithms presented in this paper belong to the first category, except one that belongs to both categories. The first exterior point simplex type algorithm (EPSA) was originally developed by Paparrizos for the assignment problem. EPSA constructs two paths to the optimal solution. One path consists of basic but not feasible solutions, while the second path is feasible. The key idea behind EPSA is that making steps in directions that are linear combinations of attractive descent direction can lead to faster convergence than that achieved by classic simplex type algorithms.

## Life sciences & healthcare

Mon.3.H 2033

### Therapy planning

Chair Laurenz Göllmann, Münster - University of Applied Sciences

Åsa Holm, Linköping University (with Åsa Carlsson Tedgren, Torbjörn Larsson)

### A new optimization model for brachytherapy dose plans

There are many types of radiotherapy, brachytherapy is one such. As a part of treatment planning, a dose plan needs to be constructed, this decides where and for how long to irradiate. Optimization of dose plans for brachytherapy is still an area that is relatively unexplored, and since the treatment is quite different, models used for external radiotherapy are not directly applicable. In this talk I will highlight the most important differences and then present the model we have formulated

and some results from our tests. Our model differs from others used in the brachytherapy field by more directly including dosimetric indices.

Rasmus Bokrantz, KTH Royal Institute of Technology / RaySearch Laboratories

### Multi-criteria optimization for volumetric-modulated arc therapy by convex decompositions

Volumetric-modulated arc therapy (VMAT) is a technique for rotational radiation therapy that has gained widespread clinical use due to its ability of improving delivery efficiency without compromising treatment quality. Treatment planning for VMAT is a challenging multi-criteria decision problem due to a high-dimensional trade-off between tumor coverage and sparing of healthy structures in the vicinity of the target volume. Here, an approach to multi-criteria VMAT optimization is presented that relies on two convex decompositions of an initially nonconvex problem formulation. An infeasible relaxation with the elements of the energy fluence vector as variables is first used to define a global trade-off between conflicting objectives. The solution to the relaxed problem is subsequently converted into a deliverable VMAT plan. A feasible restriction with segment weights as variables is finally used to evaluate deliverable solutions in its neighborhood. The practical value of the presented method is discussed in view of comparative results with a commercially available single-objective method.

Laurenz Göllmann, Münster - University of Applied Sciences (with Helmut Maurer)

### Combination therapy considered as a multiple delayed optimal control problem

We consider optimal control problems with multiple time delays in state and control and present an enhanced form of Pontryagin's minimum principle as well as a numerical discretization method.

Let  $x(t) \in \mathbb{R}^n$  denote the state and  $u(t) \in \mathbb{R}^m$  denote the control of a system at time  $t$ . Time delays for  $x$  and  $u$  are given by a vector  $(r_1, \dots, r_d)$ . The problem for two delays has been investigated earlier in [GöllmannKernMaurer09]. We now present a generalization in form of necessary conditions for the problem with multiple delays. We finally optimize a combination therapy by a model of the innate immune response with a delayed antibody production and a retarded drug action.

## Logistics, traffic, and transportation

Mon.3.H 0106

### Applications in transportation problems

Chair Paola Pellegrini, IFSTTAR - Univ. Lille Nord de France

Joshua Magbagbeola, Joseph Ayo Babalola University, Ikeji-Arakeji (with Samuel Awoniyi, Eunice Magbagbeola)

### Operations research approach to enhancing enterprise through alliances: A case study of Mowe Town, Ogun State, Nigeria

Small firm sub-sector has the potential to reduce poverty and unemployment in Nigeria. However, in the face of global competition, market uncertainties and rapid technological changes, it is necessary to assist firms, particularly small enterprise to access information that can build their business competencies to create income and employment generation opportunities. Through in-depth recourse to existing theories and empirical literature on factors that explain firm growth, the study identifies business competencies, derived through inter-firm alliances, as determinants of enterprise performance. The study establishes that the size of the firm influences the choice of business association among manufacturing enterprises in Nigeria. It is further noted that the decision to join a business association is positively related to the ages of the entrepreneur and enterprise. The study recommends incentive mechanisms that encourage business associations among small enterprise.

Hidetoshi Miura, Nanzan University (with Toshio Nemoto)

### Comparative study of reduced total travel times in check-pattern and hierarchical express systems

Express-service stop pattern on railway is an important factor to shorten travel time for long-distance users. However, it is difficult for trunk line to run enough expresses during rush hours by reason of track capacity for safety. Lack of track capacity gives trains few occasions to pass others. This study calculates reduced total travel time by expresses to compare three limited-service stop patterns: single express pattern system, check-pattern system, and hierarchical system. The hierarchical system gives stops of upper type of express to include all stops of lower expresses. The check-pattern system does not allow sharing stops between different types of expresses. Though the check-pattern system does not become common, it will give more expresses than the hierarchical system during high train density. Some simple assumptions in this railway model facilitate analytical representation to locate limited-service stops for maximizing reduced travel times. We will de-